Sequential R&D and Public Researchers' Altruism

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In this paper we extend the multisector growth model with vertical innovations in order to distinguish between basic research and industrial development. After recognizing that European patent law does not allow the patentability of scientific findings, we assume that the basic R&D is publicly funded while applied R&D is caried out by profit motivated firms. An altruistic motive for researchers is introduced. We characterize analytically and numerically the equilibrium.

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1. Introduction

The aim of this paper is to study the innovative capacity of the standard multisector neo-Schumpeterian growth model after introducing the concept of product innovation resulting from a two-stage uncertain research activity. More specifically, we explore the consequences for a quality-ladder growth model (Aghion an Howitt 1992, Grossman and Helpman, 1991) of arguing that innovation process is a follow-on-discovery process where profit-guided researchers build on the state-of-the-art public basic research level. In other words, R&D activity splits into two subsequent stages (i.e. two *half-ideas*): inventing and innovating.

According to the standard Schumpeterian paradigm R&D is an uncertain activity modelled by a Poisson process. Each (private) R&D firm employs a flow of skilled labor input z in order to obtain, un the assumption of constant return to scale, a flow probability of innovation θz, where θ is the given arrival parameter of the Poisson process.

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However a wide literature on cumulative and sequential innovation (see, for instance, Schotchemer,1991) emphasizes how in most cases the value of an idea cannot be directly embodied into the market value of a good. Think about the practice of research activity in the medical/pharmaceutical sector: once a new chemical active priciple for treating a human pathology is inviduated, a long period of pure experimental use begins in order to implement the new drug saleable to the drug market.

The contrast between the evidence of an upstream conditioned R&D activity and the conception that only the concrete embodiment of an idea is provided of economic value merges also from the increasing concern among both scholars and the business community about the ability of researchers to condut sequential R&D activity effectively (see Heller and Eisemberg, 1998 and Shapiro, 2001). In this light this paper tries to investigate the relation between the cumulative uncertainty involved in the two-stages innovation process and the inefficienciencies in the public university system. We know how in the U.S. the passage of the Bayh-Dole Act simplified the patenting of government-supported research outputs and that these last are often upstream to the development of innovative products to be sold to the market.

The sector of biotechnology offers many illustrations of the subsequential behavior of the research activity. The success in advanced biotechnology, in agricultural, and medical application fields is made possible thanks to the progress in genetic engennering allowing to transfer genetic sequences from one organism to another[3]. Since in 1973 Herbert Boyer and Stanley Cohen invented the cloning technique of genetically engineered molecules, many different applications of such technique, from erythropoietin to treat anaemia to transgenic crops, were introduced. Could one state that Boyer and Cohen's achievement is lacking in economic value just because further research needed to make it applicable for commercial purpose? What is the rationale of the behavior of publicly-hired researchers

[3]In 1980, in the Diamonds v. Chakrabarty case, the Supreme Court of United States ruled that microorganisms produced by genetic engineering could be patented. The Supreme Court's decision came two years before the introduction of the first commercial product, human insulin, made with recombinant DNA techniques.

employed in the basic research activity? Here we try to answer to this last question by introducing an altruistic motive for the publicly-hired basic researchers.

Several studies documented an increasing complexity in the applied R&D activity (Kortum 1993 and 1997; Segerstrom, 1998). If applied R&D becomes increasingly more complicated, it is important to have a large flow of half-ideas from basic research. This implies that the social awarness of the importance of the allocation of public basic R&D between the different sectors of the economy could play a central role in order to promote economic growth.

The rest of this paper is organized as follows. Section 2 sets up a Schumpeterian model with sequential innovation where basic research findings are conceived and put into the public domain, and subsequently embodied into marketable products by a large number of perfectly competitive private R&D firms. Section 3 summarizes the main results of our model.

2. The Model

2.1 Overview

Consider an economy made up of a differentiated final good sector and a differentiated research and development (R&D) sector, along the lines of Grossman and Helpman (1991), where product improvements occur in the consumption good industries. Within each industry, firms are distinguished by the quality of the final good they produce. When the state-of-the-art quality product in an industry $\omega \in [0,1]$ is $j_t(\omega)$, research firms compete in order to learn how to produce the $j_t(\omega)+1$st quality product. This learning process involves a two-stage innovation path, so first a R&D unit catches a glimpse of innovation through the $j_t(\omega)+\frac{1}{2}$th inventive half-idea and then other firms engage in a patent race to implement it in the $j_t(\omega)+1$st quality product. We rule out industrial secret and assume that, once invented, the first "half-idea" can be used by anyone to try to complete it

In what follows we refer to the term "quality leader" to denote the firm that produces the current state-of-the-art quality product. Only non-profit motivated R&D units - i.e.

public laboratories - try to invent a new first half idea in the basic research sector. We assume that R&D firms are able to instantaneously patent only the complete idea of a product innovation. Then, patent protection may determine a monopolistic position in the final good sector, and the winner of the final patent R&D race becomes the sole producer of a $j_t(\omega)+1$ quality consumption product.

Time is continuous with an unbounded horizon and there is a continuum of infinitely-lived dynasties of expanding households with identical intertemporally additive preferences. Heterogeneous labour, skilled and unskilled, is the only factor of production. Both labour markets are assumed perfectly competitive. In the final good sectors $\omega \in [0,1]$ monopolistically competitive firms produce differentiated consumption goods by combining skilled and unskilled labour, whereas research firms employ only skilled labour.

2.2 Households

Time $t \geq 0$ population $P(t)$ is assumed growing at rate $g > 0$ and its initial level is normalized to 1. The representative household's preferences are represented by the following intertemporal utility function:

$$U = E_0 \left[\int_0^{\infty} e^{gt} e^{-\rho t} u\left(t\right) dt \right], \tag{1}$$

where $\rho > g$ is the subjective discount rate and E_0 denotes the expectation operator as of time $t = 0$. Instantaneous utility $u\left(t\right)$ is defined as:

$$u\left(t\right) = \int_0^1 \ln \left[\sum_j \gamma^j d_{jt}\left(\omega\right) \right] d\omega, \tag{2}$$

where $d_{jt}\left(\omega\right)$ is the quantity consumed of a good of quality j (that is, a product that underwent j quality jumps) and produced in industry ω at time t. Assume that j is forced to assume integer values[4]. Parameter $\gamma > 1$ measures the size of the quality

[4] This assumption is common in the quality-ladder endogenous growth literature; still, in our framework, it has the meaning of explicitly stating that half-ideas discoveries do not affect consumer's utility.

upgrades (i.e., the magnitude of innovations). This formulation, the same as Grossman and Helpman (1991) and Segerstrom (1991), assumes that each consumer prefers higher quality products.

The representative consumer is endowed with $L > 0$ units of skilled labor and $M > 0$ units of unskilled labor summing to 1. Since labour bears no disutility it will be inelastically supplied for any level of non negative wages. Since initial population is normalized to 1, L and M will also equal, in equilibrium, the percapita supply of skilled, respectively, unskilled labour. Unskilled labor can only be employed in the final goods production. Skilled labour is the most versatile, being also able to perform R&D activities.

In the first step of the consumer's dynamic maximization problem, she selects the set $J_t(\omega)$ of the existing quality levels with the lowest quality-adjusted prices. Then, at each instant, the households allocate their income to maximize the instantaneous utility (2) taking product prices as given in the following static (instantaneous) constraint equation:

$$E(t) = \int_0^1 \sum_{j \in J_t(\omega)} p_{jt}(\omega) d_{jt}(\omega)\, d\omega. \tag{3}$$

Here $E(t)$ denotes percapita consumption expenditure and $p_{jt}(\omega)$ is the price of a product of quality j produced in industry ω at time t. Let us define $j_t^*(\omega) \equiv \max\{j : j \in J_t(\omega)\}$ Using the instantaneous optimization results, we can re-write (2) as

$$u(t) = \int_0^1 \ln\left[\gamma^{j_t^*(\omega)} E(t)/p_{j_t^*(\omega)t}(\omega)\right] d\omega = \tag{4}$$

$$= \ln[E(t)] + \ln(\gamma) \int_0^1 j_t^*(\omega) d\omega - \int_0^1 \ln[p_{j_t^*(\omega)t}(\omega)] d\omega \tag{5}$$

The solution to this maximization problem yields the static demand function:

$$d_{jt}(\omega) = \begin{cases} E(t)/p_{jt}(\omega) & \text{for } j = j_t^*(\omega) \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Only the good with the lowest quality-adjusted price is consumed, since there is no demand for any other good. We also assume, as usual, that if two products have the same quality-adjusted price, consumers will buy the higher quality product - although they are formally indifferent between the two products - because the quality leader can always slightly lower the price of its product and drive the rivals out of the market.

Therefore, given the independent and - in equilibrium and by the law of large number - deterministic evolution of the quality jumps and prices, the consumer will only choose the piecewise continuous expenditure trajectory, $E(\cdot)$, that maximizes the following functional

$$U = \int_0^\infty e^{-(\rho-g)t} \ln[E(t)]dt. \tag{7}$$

Assume that all consumers possess equal shares of all firms at time $t = 0$. Letting $A(0)$ denote the present value of human capital plus the present value of asset holdings at $t = 0$, each individual's intertemporal budget constraint is:

$$\int_0^\infty e^{-R(t)} E(t)dt \leqq A(0) \tag{8}$$

where $R(t) = \int_0^t r(s)ds$ represents the equilibrium cumulative real interest rate up to time t.

Finally, the representative consumer chooses the time pattern of consumption expenditure to maximize (7) subject to the intertemporal budget constraint (8). The optimal expenditure trajectory satisfies the Euler equation:

$$\dot{E}(t)/E(t) = r(t) - \rho \tag{9}$$

where $r(t) = \dot{R}(t)$ is the instantaneous market interest rate at time t.

Euler equation (9) implies that a constant (steady state) per-capita consumption expenditure is optimal when the instantaneous market interest rate equals the consumer's subjective discount rate. Since preferences are homothetic, in each industry aggregate demand is proportional to the representative consumer's one. E denotes the aggregate consumption spending and d denotes the aggregate demand.

2.3 Production

In this section we examine the production side of the economy. We assume constant returns to scale technologies in the (differentiated) manufacturing sectors represented by the following production functions:

$$y(\omega) = X^{\alpha}(\omega) M^{1-\alpha}(\omega), \text{ for all } \omega \in [0,1], \tag{10}$$

where $\alpha \in (0,1)$, $y(\omega)$ is the output flow per unit time, $X(\omega)$ and $M(\omega)$ are, respectively, the skilled and unskilled labour employment flows in industry $\omega \in [0,1]$. Letting w_s and w_u denote the skilled and unskilled wage rates, in each industry the quality leader seeks to minimize its total cost flow $C = w_s X(\omega) + w_u M(\omega)$ subject to constraint (10). For $y(\omega) = 1$, the solution to this minimization problem yields the conditional unskilled (11) and skilled (12) labour demands (i.e. the per-unit labour requirements):

$$M(\omega) = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(\frac{w_s}{w_u}\right)^{\alpha}, \tag{11}$$

$$X(\omega) = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{w_u}{w_s}\right)^{1-\alpha}. \tag{12}$$

Thus the (minimum) cost function is:

$$C(w_s, w_u, y) = c(w_s, w_u) y \tag{13}$$

where $c(w_s, w_u)$ is the per-unit cost function:

$$c(w_s, w_u) = \left[\left(\tfrac{1-\alpha}{\alpha}\right)^{-(1-\alpha)} + \left(\tfrac{\alpha}{1-\alpha}\right)^{-\alpha}\right] w_s^{\alpha} w_u^{1-\alpha}. \tag{14}$$

Since unskilled labour is uniquely employed in the final good sectors and all price variables (including wages) are assumed to instantaneously adjust to their market clearing values, unskilled labour aggregate demand $\int_0^1 M(\omega)\, d\omega$ is equal to its aggregate supply, $MP(t)$, at any date. Since industries are symmetric and their number is normalized to 1, in equilibrium[5] $M(\omega) = MP(t)$.

Letting $w_u = 1$, from equations (11) and (12) we get the firm's skilled labour demand negatively depending on skilled (/unskilled) wage (ratio):

$$X(\omega) = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) MP(t) \tag{15}$$

In percapita terms,

$$x(\omega) \equiv \frac{X(\omega)}{P(t)} = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M. \tag{16}$$

In each industry, at each instant, firms compete in prices. Given demand function (6), within each industry product innovation is non-drastic[6], hence the quality leader will fix its (limit) price by charging a mark-up γ over the unit cost (remember that parameter γ measures the size of product quality jumps).

$$p = \gamma c(w_s, 1) \Rightarrow d = \frac{E}{\gamma c(w_s, 1)}. \tag{17}$$

Hence each monopolist earns a flow of profit, in percapita terms, equal to

$$\begin{aligned} \pi &= \frac{\gamma - 1}{\gamma} E = (\gamma - 1)\frac{w_s x}{\alpha} \\ \pi &= (\gamma - 1)\frac{1}{1-\alpha} M. \end{aligned} \tag{18}$$

[5] More generally, with mass $N > 0$ of final good industries, in equilibrium $M(\omega) = \frac{MP(t)}{N}$.

[6] We are following Aghion and Howitt's (1992) and (1998) definition of drastic innovation as generating a sufficiently large quality jump to allow the new monopolist to maximize profits without risking the re-entry of the previous monopoly. Given the unit elastic demand, here the unconstrained profit maximizing price would be infinitely high: that would induce the previous incumbent to re-enter.

From eq.s (18) follows:

$$\frac{\gamma - 1}{\gamma} E = (\gamma - 1) \frac{1}{1 - \alpha} M \Rightarrow E = \frac{\gamma}{1 - \alpha} M. \tag{19}$$

Interestingly, eq. (19) implies that in equilibrium total expenditure is always constant. Therefore, eq. (9) implies a constant real interest rate:

$$r(t) = \rho. \tag{20}$$

2.4 R&D Sectors

In each industry, the R&D activity is a step-by-step process by which, first a new idea is invented and then it is used to find the way to introduce a higher quality product. First half-idea are new, non-obvious, non-tradeable, non-patentable and necessary to get to the product innovation: first half-ideas are research tools.

In order to depict a pre-1980 US normative environment and/or a current European patent regime we assume that the patent protection of the basic R&D result cannot be granted. Therefore the innovative process needs to resort to non-profit motivated R&D organizations to take place: publicly funded universities and laboratories have often been motivated by the induced scientific spillover on potentially marketable future technical applications.

Following Aghion and Howitt (1998, Ch.7), we assume that each R&D unit faces a ∪-shaped unit cost function[7]. Let $i = G, F$ denote a basic government researhc unit and an applied R&D firm respectively. Let N_i , with $i = G, F$ indicate the mass of public laboratories and, respectively, R&D firms in each R&D sector. The individual unit's Poisson process probability intensity to succeed in inventing a half-idea or completing one

[7] Assuming U-shaped R&D cost curves introduces some additional analytical complexity, but -beside being more realistic than the usual linear private R&D technologies - deliver more robust equilibria under different institutional scenarios. This renders our framework useful for additional extensions. It is interesting to point out that the square roots are not necessary: any exponent between 0 and 1 would work.

(i.e. introducing the product innovation) is $\theta_i(z_i - \phi, N_i, P(t))$, increasing and concave in $z_i - \phi \geq 0$, depending on the R&D effort z_i in excess of the fixed cost, in terms of labour input, $\phi > 0$, that each laboratory has to pay per-unit time in order to engage in the R&D race. In particular, we specify the per-unit time Poisson probability intensity to succeed for a public laboratory and a private R&D firm respectively as

$$\theta_G(z_i - \phi, N_i, P(t)) \equiv \frac{\lambda_0}{P(t)}\sqrt{\max(z_G - \phi, 0)}\left(\frac{N_G}{P(t)}\right)^{-a} \tag{21}$$

$$\theta_F(z_i - \phi, N_i, P(t)) \equiv \frac{\lambda_1}{P(t)}\sqrt{\max(z_F - \phi, 0)}\left(\frac{N_F}{P(t)}\right)^{-a} \tag{22}$$

where $\lambda_k > 0$, $k = 0, 1$, are R&D laboratory productivity constants; N_i $(i = G, F)$ represent the number of laboratories in each industry and constant $a > 0$ is an inter-unit intra-sectoral congestion parameter, capturing[8] the risk of R&D duplications, knowledge theft and other diseconomies of fragmentation in the R&D. Each Poisson process - with arrival rates described by (21)-(22) - governing the assumed two-stage innovative process is supposed to be independent across laboratories and across industries.

Eq.s (21)-(22) state that the probability intensity of the invention of a half-idea decreases with population. This assumption, common to Dinopoulos and Segerstrom (1999), captures the complexity of improving a good in a way that renders a larger population happier. Notice that also the congestion externality is assumed to decrease with population, as we deem it reasonable that the risk of R&D duplications declines with the difficulty of duplications, that the industrial espionage activities are rendered more complicated with the technological complexity of the ideas being targeted, etc. The specific form postulated for our assumption of increasing technological complexity is sufficient to guarantee that the equilibrium long run percapita growth rates do not increase with population, thereby rendering our model immune to the embarrassing strong scale effect (Jones 2003) that plagued the early generation endogenous growth models, without leading to "semi-endogenous" growth (Jones 1995, Segerstrom 1998).

[8] As, for example, in Romer's (1990) specification of the R&D technology.

From the Poisson process properties, the probability of simultaneously inventing two half-ideas in a tiny interval of time of duration Δt is a zero of order higher than the first. As a result, no industry has more than one follower and the whole set of industries $\omega \in [0,1]$ gets partitioned into two sets of industries: industries $\omega \in A_0$ (temporarily) with no half-ideas and, therefore, with one quality leader (the final product patent holder) indirectly challenged by the public R&D units, and the industries $\omega \in A_1 = [0,1]\backslash A_0$ industries, with one half-idea and, therefore, one half-idea leader (the final product patent holder) and a mass of private R&D firms aiming to complete the half-idea and to displace the current monopolist. Researchers engage in basic R&D only in $\omega \in A_0$ industries and engage in applied R&D activity aimed at a direct product innovation only in A_1 industries. When a quality improvement occurs in an industry the R&D firm that completed a half-idea becomes the new quality leader and the industry switches from A_1 to A_0. When an inventive half-idea discovery arises in an industry $\omega \in A_0$ this industry switches to A_1 and the second-stage patent race starts. Figure 1 illustrates the flow of industries from a condition to the other:

Insert Figure 1

Notice that the two sets A_0 and A_1 change over time, even if the economy will eventually admit a steady state. At any instant we can measure the mass of industries without any half-idea as $m(A_0) \in [0,1]$, and the mass of industries with an uncompleted half-idea as $m(A_1) = 1 - m(A_0)$. Clearly, in a steady state these measures will be constant, as the flows in and out will offset each other. In light of the definitions so far, we can express the skilled labor market equilibrium in percapita terms as:

$$L = x + \bar{L}_G + m(A_1) n_F z_F, \qquad \text{(L}'\text{)}$$

where $n_O \equiv \frac{N_O}{P(t)}$ and $n_F \equiv \frac{N_F}{P(t)}$ are the percapita number of laboratories in each basic and applied R&D sector, and $\bar{L}_G$.is the mass of researchers employed in the public laboratories. Eq. (L') states that, at each date, the aggregate supply of skilled labor,

$LP(t)$, finds employment in the manufacturing firms of all $[0,1]$ sectors, x, and in the R&D laboratories of the A_0 sectors, $\bar{L}_G$, and in the R&D firms of the A_1 sectors, $n_F z_F$.

We make the following behavioral rule for public researchers: we assume that public researchers can be perfectly mobile across sectors, so that when in a sector ω that lacked a half-idea, i.e. belonged to A_0, a half-idea appears, i.e. it becomes A_1, the public R&D workers canstop carrying out basic research in that sector and spread over the new A_0 set of sector. This may represent the case of university researchers who keep investigating along intellectual trajectories only when they know that private R&D firms will later profit from adapting to their market the new knowledge they may create. Unguided by the invisible hand, researchers may follow it indirectly, motivated by altruism towards society: depending on their social motivation they will choose to become more or less intellectually mobile. We will assume from here on that the public researchers are allocated across different industries according to a uniform distribution over a set of sectors that always contains A_0. The mass of this set of sectors is

$$\eta m(A_0) + (1-\eta)1 \tag{23}$$

where $\eta \in [0,1]$ measures the altruistic motivated degree of efficiency of public researchers. According to (23) if $\eta = 1$ the public researchers will target their research efforts only where society needs them, whereas if $\eta = 0$ they will be completely indifferent and will keep trying to invent second, third, etc. half-ideas even when these are redundant for the economy, but just to enrich their scientific *cv.* This behavioral assumption emphasizes the role of social awareness to help markets provide the R&D laboratories the right incentives to divert their resources from the redundant sectors and to quickly reallocate them towards more beneficial aims.

We also make the assumption that the government chooses the fraction, $\bar{L}_G \in [0, L]$, of population of skilled workers to be allocated to the heterogenous research activities conducted by universities and other scientific institutions. The government basic R&D expenditure, equal to $P(t)\bar{L}_G w_s$, is funded by lump sum per-capita taxes on consumers.

The assumption of lump sum taxation guarantees that government R&D expenditure does not imply additional distortions on private decisions. This allows us to use the previous notation and derivations also for the case of a balanced government budget taxing all households in order to transfer the tax proceeds to the basic R&D workers.

2.5 Equilibrium

The optimizing behavior of the public sector consists of maximizing the expected flow of half-ideas per sector with respect to the intensity of basic research effort z_G, that is the government chooses the optimal scale for the public laboratories.

The fixed percapita amount of skilled workers, $\bar{L}_G$, hired in the basic public R&D is equal to the intensity of basic research effort, z_G, multiplied by the number of public laboratories, N_G, i.e.:

$$P(t)\bar{L}_G = \left[\eta m(A_0) + (1-\eta)\right] N_G z_G. \tag{24}$$

In percapita terms,

$$\bar{L}_G = \frac{\left[\eta m(A_0) + (1-\eta)\right] N_G z_G}{P(t)} \equiv \left[\eta m(A_0) + (1-\eta)\right] n_G z_G. \tag{25}$$

Lemma 1 *The solution of the public sector maximization problem is* $z_G^* = 2\phi\frac{1-a}{1-2a}$.

Proof From eq.(27) we have:

$$n_G = \frac{\bar{L}_G}{z_G\left[\eta m(A_0) + (1-\eta)\right]}. \tag{26}$$

The public authorities seek to maximize the per sector- expected flow of half ideas by choosing the optimal scale for public laboratories:

$$\max_{z_G} \frac{\bar{L}_G}{z_G\left[\eta m(A_0) + (1-\eta)\right]}\theta_G\left(z_i - \phi, N_i, P(t)\right) = \max_{z_G}\left(\frac{\bar{L}_G}{z_G\left[\eta m(A_0) + (1-\eta)\right]}\right)^{1-a}\lambda_0\sqrt{z_G - \phi}. \tag{27}$$

The solution for the public sector maximization problem (26) is:

$$z_G^* = 2\phi \frac{1-a}{1-2a} \tag{28}$$

Q.E.D.

Therefore, solving eq. (25) for n_G and substituting the solution of the government maximization problem, we have:

$$n_G = \frac{\bar{L}_G}{2\phi \frac{1-a}{1-2a} \left[\eta m(A_0) + (1-\eta)\right]}. \tag{29}$$

The financial arbitrage implies the following market valuations of each sector's unchallenged[9] leader firm, V_L^0, directly challenged leader firm, V_L^1, and each R&D firm V_F[10]:

$$rV_L^0 = \pi P(t) - \left(\frac{N_G}{P(t)}\right)^{1-a} \lambda_0 \sqrt{z_G - \phi}\left(V_L^0 - V_L^1\right) + \frac{dV_L^0}{dt} \tag{30a}$$

$$rV_L^1 = \pi P(t) - \lambda_1 \left(\frac{N_F}{P(t)}\right)^{1-a} \sqrt{z_F - \phi} V_L^1 + \frac{dV_L^1}{dt} \tag{30b}$$

$$rV_F = \max_{z_F} \frac{\lambda_1}{P(t)} \left(\frac{N_F}{P(t)}\right)^{-a} \sqrt{z_F - \phi}\left(V_L^0 - V_F\right) - w_s z_F + \frac{dV_F}{dt} \tag{30c}$$

Plugging eq. (29) and the optimal size of public laboratories as given in **Lemma 3** into (30a) and using percapita variables - as in the last section - allow us to rewrite the equation of leader's financial arbitrage as:

[9] We here mean "unchallenged" by a second-stage patent race. However, a monopolist in an A_0 industry is indirectly challenged by the basic R&D laboratories trying to invent a new half-idea on which future follower firms will work to render it obsolete.

[10] Notice that $\lambda_0 \left(\frac{N_G}{P(t)}\right)^{1-a} \sqrt{z_G - \phi}$ captures the expected partial obsolescence of unchallenged leadership in each A_0 sector, $\lambda_1 \left(\frac{N_F}{P(t)}\right)^{1-a} \sqrt{z_F - \phi}$ the expected final obsolescence of directly challenged leadership in each sector A_1, and $\frac{\lambda_1}{P(t)} n_F^{-a} \sqrt{z_F - \phi}$ the probability per unit time that a single follower succeeds in each sector A_1.

$$\rho v_L^0 = \pi - \left(\frac{\bar{L}_G}{[\eta m(A_0) + (1-\eta)]\, 2\phi \frac{1-a}{1-2a}} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1-2a}} \left(v_L^0 - v_L^1 \right) + \frac{dv_L^0}{dt}. \tag{31}$$

Solving Bellman eq. (30c) and setting (as a consequence of free entry into the applied R&D sector) $V_F = 0$, we get the flow of research labor hired by each applied R&D firm, $z_F^* = 2\phi$. Hence, the previous system (30a)-(30c) can be rewritten in percapita terms as:

$$\begin{aligned}
(\rho - g) v_L^0 &= (\gamma - 1) \frac{1}{1-\alpha} M - \left(\frac{\bar{L}_G}{[\eta m(A_0) + (1-\eta)]\, 2\phi \frac{1-a}{1-2a}} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1-2a}} \left(v_L^0 - v_L^1 \right) + \frac{dv_L^0}{dt} \\
(\rho - g) v_L^1 &= (\gamma - 1) \frac{1}{1-\alpha} M - \lambda_1 n_F^{1-a} \sqrt{\phi} v_L^1 + \frac{dv_L^1}{dt} \\
0 &= \lambda_1 \sqrt{\phi} v_L^0 n_F^{-a} - w_s 2\phi
\end{aligned} \tag{32a}$$

From eq. (32a), we can solve for the skilled/unskilled wage ratio, getting:

$$w_s = \max \left(\frac{\lambda_1 v_L^0}{2\sqrt{\phi}} n_F^{-a}, 1 \right). \tag{33}$$

Let us remember that, from the final production analysis, we have:

$$x = \frac{1}{w_s} \left(\frac{\alpha}{1-\alpha} \right) M. \tag{34}$$

The dynamics of the industries is now described by the following first order ordinary differential equation:

$$\begin{aligned}
\frac{dm(A_0)}{dt} &= (1 - m(A_0))\, N_F \frac{\lambda_1}{P(t)} \left(\frac{N_F}{P(t)} \right)^{-a} \sqrt{z_F^* - \phi} - m(A_0) N_G \frac{\lambda_0}{P(t)} \left(\frac{N_G}{P(t)} \right)^{-a} \sqrt{z_G^* - \phi} = \\
&= (1 - m(A_0))\, n_F^{1-a} \lambda_1 \sqrt{\phi} - m(A_0) \left(\frac{\bar{L}_G}{[\eta m(A_0) + (1-\eta)]\, 2\phi \frac{1-a}{1-2a}} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1-2a}}
\end{aligned} \tag{35}$$

From the skilled labor market clearing condition

$$x + \bar{L}_G + (1 - m(A_0))n_F 2\phi = L, \tag{36}$$

we get to the equilibrium mass of per-sector followers:

$$n_F = \frac{L - \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M - \bar{L}_G}{2\phi(1 - m(A_0))}, \qquad m(A_0) \in [0, 1]. \tag{37}$$

In the stationary distribution $\frac{dm(A_0)}{dt} = 0$. Therefore the flow of industries entering the A_0 group must equal the flow of industries entering the A_1 group. Given the complexity of our problem, also in this case we performed numerical simulations in Matlab[11]. In all simulations a unique economically meaningful steady state equilibrium exists and it is determinate.

Moreover, in all numerical simulations we observe a higher steady state innovation rate and welfare the higher the degree of altruism η.

3. Final Remarks

This paper developed a general equilibrium R&D-driven growth model in which the innovation process is decomposed into two successive innovative stages. The extension of multisector Schumpeterian models to such a more realistic dimension allows us to depict a stylized version of the current European technological institution and to set the framework for an answer of the question about a potential US-orented policy shift towards the extension of patentability to research tools and basic scientific ideas. These normative innovations have been modifying the US industrial and academic lives in the last two decades. However, we emphasize the role of the public researcher's need for achievement of useful intellectual discoveries in positively affecting long run percapita utility growth rates.

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[11] The files .mod used to simulate the model in Matlab are available from the authors on request.

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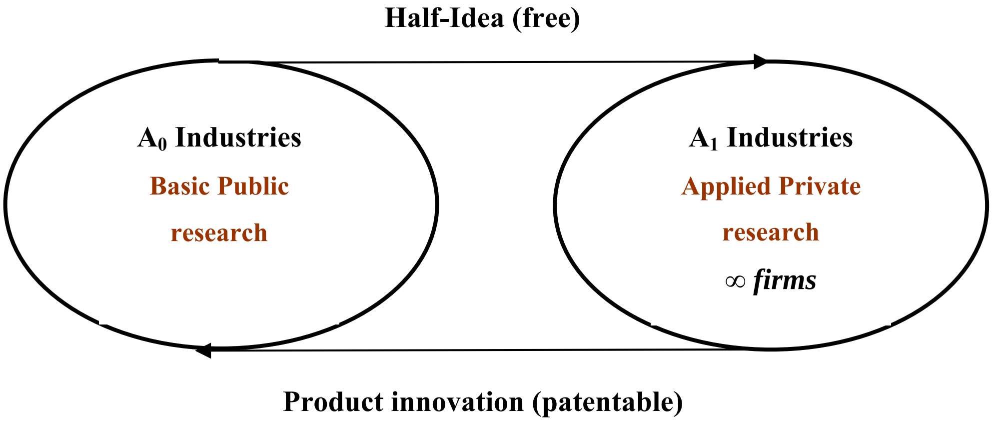

FIGURE 1: REPRESENTATION OF THE ECONOMY BY FLOWS OF INDUSTRIES.