Entry and stationary equilibrium prices
in a post-keynesian growth model

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Abstract. A peculiar feature of the post-keynesian growth theory is its focus on the growth objective of the firm and the price implications of this objective (see, e.g. Kregel and Eichner (1975), Shapiro (1981)). The post-keynesian view of price formation, as canonically represented by the mark up theory, is essentially different from the neoclassical one as the prices are essentially long period prices and they reflects the reproducibility of the firm's activity.

The existing theories of formation of mark-up within a post-keynesian context is essentially a partial equilibrium theory (see e.g. Steindl (1965), Eichner (1975), Shapiro (1981)). The aim of this paper is to develop a simple model of mark up formation driven by the threat of entry, very close in spirit to Steindl and Eichner's approach but within a full-fledged post-keynesian growth model. We shall show that if the wage rate is given, then the attainment of the full employment steady state can be impeded by firms'coordination failures in setting prices. We show also that in order to ensure a full employment growth there is room for coordinating government policies, which essentially amounts to ensure an "optimal" degree of competition in the economy. If we allow flexibility of the wage rate, the automatic adjustment of the economy towards the full employment steady state as conceived by Kaldor can be recovered, although, unlike his view, in this case the labour market acts as a coordinating device.

J.E.L. Classification Numbers: B22, E12, L20, O41.

1. Introduction

The recent theory of endogenous growth has revitalised the neoclassical growth theory by showing that it is able to deal with increasing returns and imperfect competition (see, e.g., Aghion and Howitt (1998)). As an alternative paradigm, the post-Keynesian theory is a much more rich theory than the neoclassical growth theory as its has a strong empirical ring and reasonable assumptions (see Kaldor (1956), (1957), Pasinetti (1962)): first, the post-Keynesian growth theory is an intrinsically dynamic theory which accounts for uncertainty and expectations; second, it allows for the distribution of income to have a role in the determination of the economic activity of an economy; thirdly it recognises the fundamental role of money; finally, it proposes a pricing theory which is much more realistic with respect to the neoclassical one based upon the profit-maximizing behaviour of firms over infinite horizons (Eichner and Kregel (1975)).

The role played by distribution of income in the post-keynesian growth theory makes the pricing of firms one of the most important aspects of this theory. Actually, the view elaborated by the post keynesian theory renders this theory particularly original with respect to neoclassical one as, according to it, prices have not any allocative role, their main role it being to ensure the reproducibility and the expansion of the economic system (Eichner (1981, p. 81)). This makes the post-keynesian price theory very close to the idea of production price held by Classical economists like Smith (1975) and Ricardo (1951), and by the more recent work by Sraffa (1960) (*Ibidem.*). Moreover, the importance of the role of pricing within the post-keynesian growth theory has been testified by the very many works that have been produced on this topic (for a rather complete list see Lee (1998)). According to us, however, all attempts made by this theory to insert a sound theory of the pricing behaviour of firms into a growth model have the major shortcoming of being carried out within a partial equilibrium analysis. Hence, it remains to make clear to what extent the post-keynesian price theory is coherent with its view of the growth process.

Within the keynesian tradition, the recent stream of literature called new keynesian economics has provided an important contribution to the construction of a theory in which the most important keynesian propositions are obtained on the basis of static general equilibrium macroeconomic models with a solid microeconomic foundation (see, e.g. Stiglitz (1992), Mankiw and Romer (1991), Silvestre (1993), Benassi, Chirco and Colombo (1994), Dixon and Rankin (1995)). One of the main features of this approach is the relaxation of the assumption of perfect competition; a natural consequence of this theoretical step, which is highly justified from the empirical point of view, is the emergence of macroeconomic externalities and strategic complementarities which, in turn, are used to explain the rigidity of prices and, therefore, to ensure the possibility of

underemployment equilibrium configurations even in presence of potentially flexible prices and real wage rate (see, e.g. Cooper (1999)).

The aim of this paper is to provide a simple explanation of the formation of price (and of the distributive shares) in a post-Keynesian growth model. The advantage of our approach is that it is based upon assumptions which are compatible with the post-Keynesian approach as far as the behavioural hypothesis on firms and the role of saving behaviour of households and firms in determining the steady state are concerned. Moreover, as our pricing mechanism is based upon the competitive pressure determined by the threat of entry, our model links post-Keynesian growth theory with the classical view of price determination as far as the role of potential competition is concerned, and with the new keynesian economics as far as the role of imperfect competition is concerned in explaining the possibility of existence of underemployment (dynamic) equilibria.

The paper is organized as follows: in the next section a summary of the price theory in the post-keynesian growth theory and a brief restatement of the entry game within our perspective are presented. Section 3 develops a post-Keynesian growth model where the distribution is determined according to the that entry game. We show that there is room for steady state equilibria with underemployment and that this situation is due to coordination failures in setting prices.[1] Section 4 takes up the factors yielding full employment. Final conclusions are contained in Section 5.

2. Pricing theory and steady state equilibrium in post-Keynesian theory[2]

The neoclassical theory bases its pricing theory of goods and factors on the profit-maximizing behaviour of firms and on the role of market in determining prices (see, e.g. Marshall (1869)). By contrast, the post Keynesian theory of growth and distribution closely follows the "full cost" pricing introduced by Hall and Hitch (1939) as an alternative approach to the firm's behaviour. The theory of "full cost" rejects the idea that firms maximize their profits by equating marginal costs to marginal revenue, leaving to the market the determination of the price; by contrast, according to this theory price is set equal to the "full average cost (including a conventional allowance for profits)" Hall and Hitch (1939, p. 17)). Two problems arise here: first, how to compute the "full cost", second to explain how the "conventional" profit margin is determined. As for the former problem, Hall and Hitch themselves point out that there are different ways of computing "full costs", surely

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[1] Osano (1989) develops a growth model in which coordination failures by firms and workers in the accumulation process of knowledge can trap the economy into a low steady state growth rate.
[2] For an in depth account of post-Keynesian pricing theory the reader is referred to Lee (1998)

not necessarily determined by input prices and technology (Hall and Hitch, 1939, p. 17-18). As for the latter one, they simply state that the profit margin is determined by competition, whose role is to "induce firms to modify the margin of profits which could be added to direct costs and overheads so that approximately the same prices for similar products would rule within the 'group' of competing producers" (*Ibidem.*). Moreover, "the conventional addition for profits varies from firm to firm and even within firms for different products" Hall and Hitch (1939, p. 17)). A second major step toward an alternative view of the pricing behaviour is taken by Hall and Hitch by saying that prices are not determined by short-period market contingencies but by the long run conditions of survival of the firm in the industry, in particular the entry of new firms in the industry (Hall and Hitch (1939, p. 22, point (v))). Interesting as it may be, Hall and Hitch's "full cost" pricing theory lacks of a precise explanation of the way in which the profit margin is determined.

As said above, with its emphasis on the necessity to construct a theory of firms behaviour within a long period context, the post-Keynesian theory of growth and distribution has heavily recovered the full cost theory to deal with the pricing behaviour of firms. This methodological move justifies the rejection of the profit-maximization assumption by firms, as it is only a short-period rational behaviour, it implying that agents do not worry about the future (Shapiro and Mott, 1995, p. 40). If long period survival should be taken into account, then firms have to protect their markets by avoiding entry, so their pricing choices must essentially be based upon a long period horizon, i.e. it must be strategic rather then profit-maximizing. With its essentially forward looking and long period nature, the full cost pricing rule provides the correct tool to analyse the long run behaviour of firms. In fact, its determination includes the recouping of costs and takes into account the planned growth of firm (Shapiro and Mott, 1995, pp. 40-1, see also Shapiro and Sawyer (2003)). As a matter of fact, within the "full costs" pricing theory, in fact, growth is the most important target for the firm because growth is a necessary condition for survival: "the market share has to be protected –its market power and security rest on it – and market protection requires investment. The output capacity of the firm has to expand in step with the market" (Shapiro and Mott, 1995, pp. 40-1). Actually, according to Kaldor the survival of the firm in the long run is ensured if the firm grows as much as possible since the "competitive strength" of a firm depends upon the share of market (Kaldor (1966, p. 67)). This view makes the post-Keynesian conception of price an *ex ante* variable and its view of cost a long period one.[3] Moreover, it seems to be compatible with and close to the managerial theory of firm (see Marris (1963), (1964)), according to which firms maximize the rate of growth of sales (Eichner (1973, p. 1196)).

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[3] There is a quite strong empirical evidence that prices are determined according long period (normal) costs. See, e.g. Godley and Nordhaus (1972), Coutts, Godley and Nordhaus (1978), Sawyer (1983).

Within this theoretical framework two approaches can be detected as far as the pricing question is concerned: the former, which refers to Keynes' *Treatise on Money* (1930) as principal inspiration, bases its view upon the assumption of full employment condition: if investment are greater than savings, then there is excess of demand with respect to the productive capacity at the normal price. Hence, price rises in such a way to ensure that savings equals investment (see Dutta (1990)). The mark-up is determined endogenously in such a way to yields the equilibrium condition investment equal to savings. According to the latter approach, firms operate below their full capacity and they can freely choose their mark up. The mark up is determined on the basis of either firms' "monopoly power" (see, e.g., Kalecki (1971)) or firms' requirement for financing investment (Kregel (1971), Harcourt (1972), Eichner (1973), (1975), Eichner and Kregel (1975), Wood (1975), Kaldor (1966), Harcourt and Kenyon (1976)). The kaleckian version of the mark up approach, which is based upon the elasticity of demand, does not seem to be really different from the neoclassical approach,[4] and, actually, mainly in its more recent contributions, attempts to determine endogenously the profit mark up have been accomplished using neoclassical analytical tools (see, e.g. Cowling (1981)). Moreover, this approach determines the mark up *ex post* rather than *ex ante*, the latter being the peculiar feature of the post Keynesian pricing theory, as already said (Downwards and Reynolds (1996)). On the other hand, the financing motive approach, which originates from Robinson's early work on the accumulation of capital (Robinson (1952), see also Lee (1998, p. 174)), and which is closer to the "full cost" theory of pricing, seems to be a real departure from the neoclassical maximization approach. According to Kregel (1971) and Harcourt (1972), the mark up is determined by the investment needs for ensuring the normal rate of capacity utilization, as it is the availability of finance which permits investment plans to be implemented. Within this framework, Steindl (1952) is one of the first economists who explicitly introduces the financing motive in determining the mark up, while Kaldor (1966) later considers explicitly this motive for ensuring the steady state equilibrium in his growth model. Wood (1975) argues that business enterprises aim to maximizing the rate of growth of sales revenue subject to three constraints: growth of demand, capacity growth and availability of finance (Wood (1975, Chapter 3)). After expressing the objective function and the constraints in terms of the profit margin, the latter is determined as the solution to a standard programming problem. Eichner improves upon Wood's contribution by developing a theory of the mark up product-specific rather then enterprise specific (Lee (1998, p. 181)). Following a price leadership model, also Eichner concludes that in steady state the mark up is

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[4] The interpretation according to which Kalecki's pricing theory is essentially based upon a neoclassical approach is held, for example, by Kaldor (1956, p. 92), Basile and Salvadori (1984/85), Carson (1990). For a different view, see Kriesler (1988), Downwards and Reynolds (1996).

sufficient to generate the profits required for financing the steady growth of firm.[5] According to Eichner, the mark up is determined by the equalization of the demand of additional investment fund with the supply of additional investment funds (Eichner (1973, 1975)), where the latter is constructed in such a way to include three constraints binding the pricing decision of firms: the substitution effect, the entry threat of new firms and the government intervention (Eichner (1973, p. 1190)). [6]

Although Eichner himself claim that the analysis he proposes is a contribution to the endogenous determination of the mark up within the post-Keynesian framework (Eichner (1973, p. 1196), Eichner and Kregel (1975)), it turns out that this theory is not integrated into a general equilibrium growth model, as it maintains essentially a partial equilibrium nature. In fact, the demand for investment funds is given exogenously and there is no systematic analysis of the three major constraints to firms' pricing behaviour, i.e. substitution effects, entry and government intervention. On the other hand, while Wood (1975) develops his profit theory within a macroeconomic model *à la* Harrod-Domar, his analysis of competition is *deliberately* quite vague as it being considered a typically microeconomic phenomenon. In Wood's analysis competition's role is just to make the model determinate via the financial frontier (Wood (1975, Chapter 4, Section 4.1.)).

We believe that while, on the one hand, it is important to spell out in details the competitive process in order to test the consistency of the post-Keynesian growth theory with well accepted theories of long period competitive behaviour, on the other hand, we believe that it is worth carrying out this analysis by deepening Eichner's role of entry in determining the pricing behaviour of firms. By carrying out this research programme, we should be able to link two streams of long period theories, i.e. the post-Keynesian theory of growth, according to which distribution is important in determining growth, with the classical theory of price and distribution (see e.g. Smith (1975), Ricardo (1951)), according to which entry process is the main factor determining long period prices. On the other hand, by abandoning the hypothesis of perfect competition in order to obtaining the possibility for unemployment (steady state) equilibria via the existence of macroeconomic externalities and strategic complementarities, our analysis is able to link the post-keynesian growth theory with the new keynesian macroeconomics as well.

In order to incorporate entry into a post-Keynesian growth model, we have to adapt the traditional view of the entry process, since it is usually developed within a static, partial equilibrium

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[5] Eichner develops his theory by referring to desired investment per planning period rather than to steady states. However, the two concepts are compatible as Eichner himself is quite explicit in considering his theory complementary to Marris's managerial theory of firms where a comparative dynamics approach is adopted and firms plan according to the steady state rate of growth (Marris (1969, p. 192)(1964, Chapters 3 and 8)).

[6] The crucial role of potential competition in determining the mark up is an idea as old as the post-Keynesian theory itself. See Kaldor and Robinson (2000, p. 269).

framework. Within this framework, under the assumption that there are no barriers to entry, it is usually assumed that firms enter the industry if the post-entry price yields extra-profits, i.e. a rate of profits greater then the "normal" one. By "normal" rate of profit is meant (the uniform) rate of profits which can be earned in any other industry. In a multisectoral dynamic context like ours, in which firms want to maximize their growth rate and profits are a major source of financing, we revise the above entry model by following the classical view of price formation and Eichner's approach and by taking into account the particular firms' objective function here adopted. Thus assuming that there are no barriers to entry, we suppose that firms will enter an industry if the post entry price yields profits and, consequently, savings which allow entrant firms to growth at a rate greater than the current growth rate in the other sectors.[7] Therefore, the price the incumbent firms have to choose to bar entry is that price below which the entrant firms enjoy a rate of growth lower than the rate of growth in the other sectors. [8,9]

In the following sections, we shall develop a two-sector post-Keynesian growth model in which the role of potential competition is spelled out in a more detailed way by following the entry process of classical tradition and later recovered by Eichner. We show that the (uniform) rate of growth of the economy can be explained in a steady state equilibrium by the entry preventing behaviour of incumbent firms, and that this rate of growth is not necessarily equal to the rate of growth of population (plus capital depreciation) because of the existence of strategic complementarities between the two sectors. This situation leads to a coordination failure by firms in setting prices which, in turn, yields that the economy can be trapped into a underemployment stationary equilibrium. It will be shown also that there is room for government intervention policy for diving the economy towards a full employment stationary equilibrium and that this result can be ensured also by adjustment of the real wage, which in this case has the role of a coordinating device.

Our model can be considered the dynamic two sector version of the traditional classical multisectoral model *à la* Sraffa (1960), in which the uniformity of the rate of profit is the outcome of the competitive entry process and, because of this uniformity, its level can be determined once the wage rate is known. In our model, the entry prevention behaviour of firms ensures a uniform rate of growth between the sectors and the latter is determined, once again, by the wage rate. Clearly, our model can be easily extended to an economy with three goods or more.

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[7] This assumption seems to be compatible with the view expressed by Shapiro on the factors determining entry (see Shapiro (1981, p. 92)).

[8] Ong's entry barring price is a static concept, while the price set by the incumbent firm has a long period nature (Ong (1981, p. 105)). This weakness is removed in our approach as all prices are long period ones.

[9] Following Ong's terminology, the barring entry price is a "defensive target pricing strategy" implemented by firms in an entry game (Ong (1981, p. 103)).

3. The model

Consider an economy with two sectors, indicated by 1 and 2; technology has fixed coefficients and in sector i ($i = 1,2$) it is represented by the triplet (y_i, k_{i1}, k_{i2}) where y_i indicates the amount of good i produced by using one unit of labour while k_{ij} ($j = 1,2$) indicates the pro capite amount of good j used up to produce good i. We assume also that $y_i - k_{ii} > 0$ and that $(y_1 - k_{11})(y_2 - k_{22}) - k_{12}k_{21} > 0$. This assumption means that the technology is viable, i.e. that it is able at least to reproduce itself. It is a very common assumption adopted by the literature on multisectoral growth (see e.g. Lippi (1979), Kurz and Salvadori (1995)). Finally, it is assumed that there is no technical progress and that the depreciation rate of capital in either sector is equal to zero.

Following the approach that comes back to classical economists, we assume that the nominal wage rate w is uniform across sectors and that population grows at rate n.[10] Finally, following the institutional approach to saving behaviour (see e.g. Kaldor (1966), Pettenati (1967)), we assume that *companies* save fraction s_c of their profits while households save fraction s_w of wages, where $0 \le s_w < s_c \le 1$. For the sake if simplicity, we shall assume moreover that $s_w = 0$. This assumption implies that the accumulation process is obtained only out of profits (see, however, footnote 12). Following the post-keynesian theory based upon the financing motive, we assume that the accumulation of firms is based only upon internal financing and, therefore, upon their savings.

As far as the working of markets and the behaviour of firms are concerned, we follow the approach proposed in the preceding section characterised by the condition that there are no barriers to entry, that firms set their prices (and, therefore, the mark-up) and that the incumbent firms want to maximize their rate of growth under the condition that entry is barred in their sector, the latter condition being ensured only if the post-entry rate of growth in the relevant sector is lower than the growth rate in the other sector. While the growth maximising condition is close to the post-Keynesian view (see e.g. Marris (1969), Eichner (1975), Wood (1975)), the entry-preventing behaviour – which rules out perfect competition – makes it explicit the interdependence amongst firms and amongst sectors, interdependence which is a further peculiar feature of post-Keynesian microeconomic theory (Eichner and Kregel (1975, p. 1305)), and which can be traced back to the classical tradition of Smith and Ricardo.

Let us indicate by π_i and p_i the pro-capite mark up in sector i and the price of good i. We obtain the price equation in sector i:

$$p_i y_i = \pi_i + p_1 k_{i1} + p_2 k_{i2} + w. \qquad (1)$$

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[10] It is obvious that the rate of population growth can depend upon the wage rate. For the sake if simplicity, we shall neglect this possibility.

In order for sector i to grow at rate n_i, it is necessary that the pro-capite savings generated in that sector are equal to the pro-capite effective depreciation rate at growth rate n_i;[11] i.e.

$$s_c \pi_i = n_i(p_1 k_{i1} + p_2 k_{i2} + w) \qquad . \qquad (2)$$

From (1) and (2) we obtain the following fundamental relationships:[12]

$$n_1 = \frac{s_c p_1 y_1}{p_1 k_{11} + p_2 k_{12} + w} - s_c . \qquad (3.1)$$

$$n_2 = \frac{s_c p_2 y_2}{p_1 k_{21} + p_2 k_{22} + w} - s_c \qquad (3.2)$$

In the remaining part of this section we shall use expressions (3.1) and (3.2.) in order to define a relationship between the prices and the growth rates, given the wage rate. In the next section we shall allow the wage rate to change. In general, the relationship between the growth rate in sector i on the one hand and the prices and wage rate on the other will be denoted by $n_i(p_1, p_2; w)$. From (3.i) it is possible to see that n_i is a differentiable function of prices and wage rate; moreover, for every non negative triplet $(p_1, p_2; w)$: $n_i(0, p_2; w) = -s_c$, $n_i(p_1, 0; w) > -s_c$, $n_i(p_1, p_2; w) \to \alpha > 0$ as $p_i \to \infty$, $n_i(p_1, p_2; w) \to \beta < 0$ as $p_j \to \infty$, $\partial n_i(p_1, p_2; w)/\partial p_i > 0$, $\partial n_i(p_1, p_2; w)/\partial p_j < 0$ (for these properties, see *Fact A* in the Mathematical Appendix).

Consider now a given positive price p_j and choose any positive price p_i such that entry is barred in sector i; i.e. which satisfies the condition $n_i(p_1, p_2; w) \le n_j(p_1, p_2; w)$. By the mathematical properties of functions $n_i(p_1, p_2; w)$ and $n_j(p_1, p_2; w)$ given above, it follows that for every $p_i' < p_i$ one obtains: $n_i(p_1', p_2; w) < n_i(p_1, p_2; w) \le n_j(p_1, p_2; w)$, i.e. entry is barred in sector i. By the behavioural assumption of firms it follows that, given p_j, firms in sector i will set a price which is solution to the following programme:

$\max_{p_i} n_i(p_1, p_2; w)$, subject to $n_i(p_1, p_2; w) \le n_j(p_1, p_2; w)$. (4.$i$)

Again, from the mathematical properties of the growth rate functions, it follows that for every positive p_j, there exists a unique and positive solution $p_i^*(p_j; w)$ to programme (4.i) (for positivity, see *Fact B* in Mathematical Appendix). Moreover, this solution satisfies the constraint as an equality; i.e. the growth rates in either sectors must be equal. From this, it follows immediately that, for example, given $p_1^* = p_1^*(p_2; w)$, price p_2 is the solution to programme $\max_{p_2} n_2(p_1^*, p_2; w)$, subject to $n_2(p_1^*, p_2; w) \le n_2(p_1^*, p_2; w)$. For a graphical illustration of this solution yielding a

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[11] By effective depreciation is meant the reduction over time of the pro-capite capital due to wear and tear of the physical capital (i.e. physical depreciation of capital which is assumed to be zero) plus the reduction of the pro-capite capital due to the growth of population.

[12] If $s_w > 0$, relations (3.i) becomes: $n_i = \dfrac{s_c p_i y_i + s_w w}{p_1 k_{i1} + p_2 k_{i2} + w} - s_c$. The reader can easily develops a parallel analysis for this case.

positive growth rate $n_i*(p_j; w)$ see Figure 1 (the conditions ensuring the positivity of the growth rate are analysed later).

The above argument can be formalised as a price game played by the firms in the two sectors, where firms in sector i choose the price which is solution to programme (4.i), given the price of sector j. Thus, given the properties of functions $n_i(p_1, p_2; w)$, set $P(w) = \{(p_1, p_2) \in \mathfrak{R}^2_+ |\ n_1(p_1, p_2; w) = n_2(p_1, p_2; w)\}$ is the set of price configurations which are Nash equilibria of the preceding price game. In fact, take any $(p_1, p_2) \in P(w)$, then for every price $p_i' > p_i$ ($p_i' < p_i$) one has that $n_i(p_1', p_2; w) > n_1(p_1, p_2; w)$ ($n_i(p_1', p_2; w) < n_1(p_1, p_2; w)$), therefore p_i is the price which maximises the growth rate in sector i under the condition of barred entry in that sector; i.e. it is the solution to problem (4.i). The same interpretation can be given to price p_j, given price p_i.
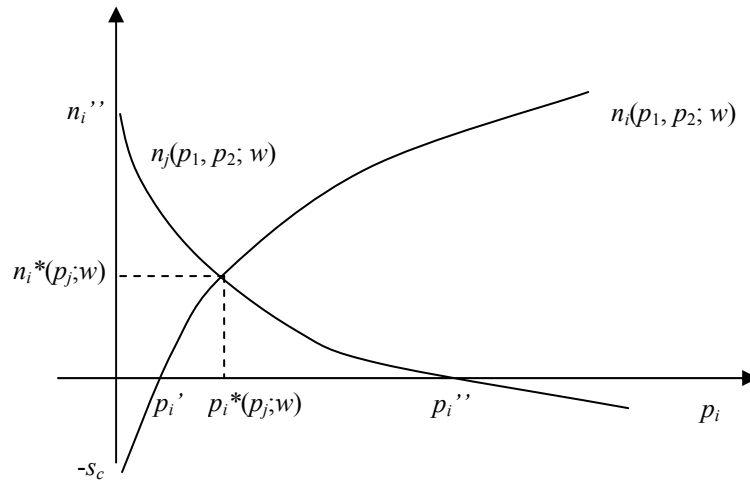
$p_i' = (p_j k_{ij} + w)/(y_i - k_{ii})$, $p_i'' = (p_j(y_j - k_{jj}) - w)/ k_{ji}$, $n'' = s_c((p_j(y_j - k_{jj}) - w)/(p_j k_{jj} + w))$

Figure 1

What has been said until now justifies the following definitions: a *stationary state* is a price configuration $(p_1*, p_2*) \in P(w)$. It is obvious that in a stationary state the rate of growth in sector 1 must be equal to the rate of growth in sector 2; denote by $n*$ the uniform rate of growth at a stationary state. A stationary state with full employment is a stationary state in which $n* = n$.

Notice also that $P(w) = \{(p_1, p_2) \in \mathfrak{R}^2_+ |\ \Phi(p_1, p_2; w) \equiv n_1(p_1, p_2; w) - n_2(p_1, p_2; w) = 0\}$ and that $\partial\Phi(p_1, p_2; w)/\partial p_1 > 0$ and $\partial\Phi(p_1, p_2; w)/\partial p_2 < 0$ for every $p_1, p_2 > 0$ (see *Fact C* in the Mathematical Appendix). Hence, by the global version of the Implicit Function Theorem (see e.g. Glustoff (1976)), it is possible to find a differentiable function P_i such that, for every positive vector $(p_j; w)$ it

holds true that $p_i*(p_j; w) = P_i(p_i; w)$ and $dP_i(p_i; w)/dp_j = - \partial\Phi(p_1, p_2; w)/\partial p_j/\partial\Phi(p_1, p_2; w)/\partial p_i > 0$, where the inequality follows from the properties of function Φ already pointed out.

From now on we shall consider only stationary states, hence configurations with a uniform rate of growth. Relationships (3.1) and (3.2.) define the (uniform) rate of growth as a function of the price vector. Inversely, we can define the price vector as a function of the (uniform) rate of growth. This relationship can be made more explicit by expressing relations (3.1.) and (3.2.) as follows:

$$A(n_1, n_2)p = b(n_1, n_2)w, \qquad (5)$$

Where $A(n_1, n_2) = \begin{bmatrix} s_c(y_1 - k_{11}) - n_1 k_{11} & -(s_c + n_1)k_{12} \\ -(s_c + n_2)k_{21} & s_c(y_2 - k_{22}) - n_2 k_{22} \end{bmatrix}$, $p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$, $b(n_1, n_2) = \begin{bmatrix} s_c + n_1 \\ s_c + n_2 \end{bmatrix}$.

Denoting by n' a generic uniform rate of growth, by the already stated assumption on technology ($y_i - k_{ii} > 0$ and $\det A(n', n') = (y_1 - k_{11})(y_2 - k_{22}) - k_{12}k_{21} > 0$), by continuity, it follows that there exists a growth rate $n^* > 0$ such that for every $n' \in (-n^*, n^*)$ the matrix $A(n', n')^{-1}$ exists and has positive entries[13] (see, for example, Kurz and Salvadori (1995)). Thus, for every $n' \in (-n^*, n^*)$ the price vector is a vector function (with non negative values) of the growth rate:

$$p(n') = A(n', n')^{-1}b(n_1, n_2)w, \qquad (6)$$

where $A(n_1, n_2)^{-1} = \dfrac{1}{\det A(n', n')}\begin{bmatrix} s_c(y_2 - k_{22}) - n'k_{22} & (s_c + n')k_{12} \\ (s_c + n')k_{21} & s_c(y_1 - k_{11}) - n'k_{11} \end{bmatrix}$. It can easily be shown

that each entry of the matrix $A(n', n')^{-1}$ is directly related to n', thus $p(n')$ is directly related to n' as well. Define the set $P(n^*, w) = \{ p \in \Re^2_+ | p = A(n', n')^{-1}b(n_1, n_2)w, n' \in (-n^*, n^*)\}$. Set $P(n^*, w)$ is the image of the price mapping defined by (6) as n' runs over the interval $(-n^*, n^*)$ and it is the subset of $P(w)$ associated to growth rates in the previous interval. This set is illustrated in Figure 2. It is increasing because, as already said, $dP_i(p_j; w)/dp_j > 0$. Moreover, prices on curve $P(n^*, w)$ are associated with an increasing uniform rate of growth as we move to the north-east direction (see Figure 2).

In Figure 2 $(p_1^0, p_2^0) = p(0)$ and $(p_1^{n'}, p_2^{n'}) = p(n')$ for $0 < n' < n^*$. By what has been said above concerning the relationship between the rate of growth n' and the associated price vector $p(n')$, it follows that $0 < p(0) < p(n') < p(n^*)$. Moreover, it is immediate to check that for every (p_1, p_2) belonging to the area A one has: $n_1(p_1, p_2; w) > n_2(p_1, p_2; w)$ and entry would occur in sector 1, while for every (p_1, p_2) belonging to the area B one has: $n_1(p_1, p_2; w) < n_2(p_1, p_2; w)$ and entry would occur in sector 2, while every $(p_1, p_2) \in P(n^*, w)$ is a stationary equilibrium.

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[13] More specifically $n^* = (\lambda_{(n',n')}^{-1} - 1)s_c$, where $\lambda_{(n',n')}$ is the dominant eigenvalue of matrix $A(n', n')$.
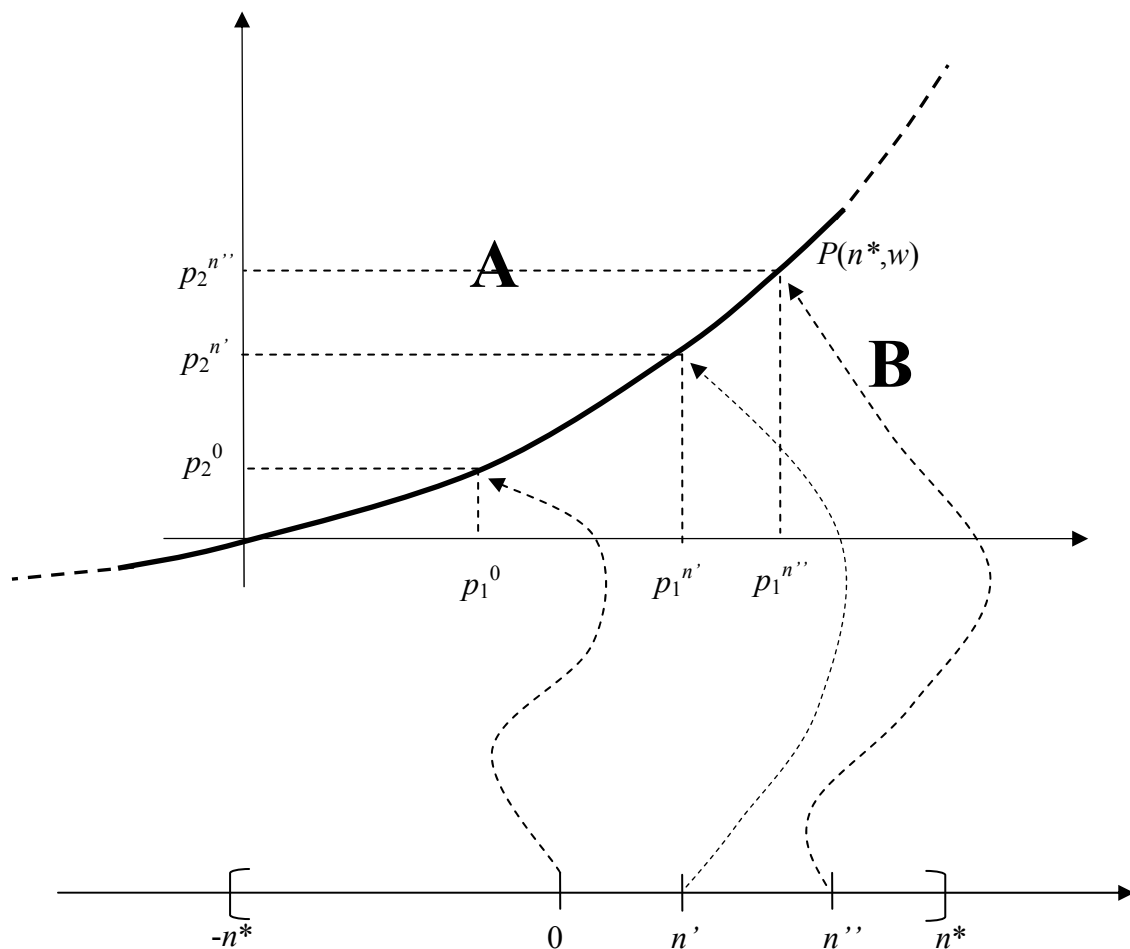
Figure 2

It should be clear that nothing ensures that the rate of population growth $n \in (-n^*, n^*)$. This happens if $s_c(y_1 - k_{11}) - nk_{11} > 0$, $s_c(y_2 - k_{22}) - nk_{22} > 0$, and $\det A(n, n) > 0$. In fact, if these assumptions are satisfied, then they are satisfied for every $0 < n' < n$. Moreover, by continuity we can take $n^* > n$. Notice that at the price vector $p(n)$ the economy is growing at the rate of growth of population.

However, even if we assume that $n \in (-n^*, n^*)$, nothing ensures that the economy will attain $p(n)$. In fact, at any price vector $p(n') \in P(w)$ with $n' \neq n$, there is no incentive by firms to change unilaterally their prices, given the price in the other sector. More specifically, if $n' < n$, then at price $p(n') \in P(w)$ the sectors are growing at a uniform rate n', hence there is an increasing unemployment. Anyway, there is no incentive by firms to change their prices to prices $p^n = (p_1^n, p_2^n)$. In fact, if, for example, firms in sector 1 would increase their price from $p_1^{n'}$ to p_1^n, then the price vector would be in area A in Figure A and entry would be profitable in sector 1. However, firms in sector 1 would find it profitable to increase their price from $p_1^{n'}$ to p_1^n if, at the same time, firms in sector 2 increase their price from $p_2^{n'}$ to p_2^n. If this is the case, the economy would shift

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from a stationary equilibrium with increasing unemployment to a stationary equilibrium in which the rate of growth of the economy is equal to the population growth rate.

From the previous argument, it should be clear that the economy can be trapped into a stationary equilibrium with unemployment because of a coordination failure of firms in setting prices (see e.g. Cooper and John (1988), and references therein; see also Benassi, Chirco and Colombo (1993), Boitani and Delli Gatti (2001)). In fact, following the terminology of the literature on coordination failure, there is strategic complementarity between the two sectors as an increase of the price of a sector yields an increase in the best price (i.e. the entry barring one) of the other sector. Moreover, by the properties previously mentioned concerning price vectors in $P(w)$, it follows that such equilibria can be Pareto ranked in terms of the growth rate, where, according to the assumed behaviour of firms, the optimum (long period) rate of growth could be n.[14] Finally, in our model, like any classical, neoclassical and endogenous growth model (see e.g. D'Agata and Freni (2003)), Say's Law holds because there is always coincidence between aggregate supply and aggregate demand, as firms reinvest all their savings; however, in our model Say's Law is no longer "equivalent to the proposition that there is no obstacle to full employment"(Keynes, 1936, p. 56; quoted from Boitani and Delli Gatti (2001, p. 419); but, on this interpretation, see Baumol (1999)). This is the origin of the possibility of the underemployment trap described before.

4. Stationary state with full employment

Following again the logic of the literature on coordination failures, the problem of the selection of (optimal) equilibrium arises, i.e. the selection of price vector $p(n)$. To this problem several solutions have been proposed, and they usually rely on Pareto-improving government policies (see e.g. Cooper (1994, 1999), Benassi, Chirco and Colombo (1993) Boitani and Delli Gatti (1991, 2001)). Within our context, a simple selection mechanism could be a regulatory intervention by the government in order to ensure the choice of the "right" price $p(n)$. The mechanism could work as follows: the government protects incumbent firms in sector i from entry if the current price p_i is not higher that p_i^n; by contrast if $p_i > p_i^n$ the government would favour entry by incentivating the creation of new firms or entry by foreign firms. This intervention would amount to ensure the

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[14] Given the behavioural hypothesis on firms, it is reasonable to assume that in ranking growth rates firms refer to *feasible* long period growth rates. Actually, n is the highest rate of growth which firms can sustain in the long period.

attainment of the "right" level of (lack of) competition, as measured by the level of price or of the associated mark-up.

An alternative solution is the attainment of a full employment stationary equilibrium via a change of the nominal wage. This solution is close to the Kaldor's view of the adjustment toward the full employment dynamic equilibrium, although here, as will be clear later, the labour market plays the role of coordination mechanism, which is quite different from its equilibrating role in Kaldor's analysis.

Let us assume that the adjustment of prices is instantaneous with respect to changes of the wage rate, and prices react to wage rate changes in such a way to ensure instantaneously a uniform rate of growth between sectors, according to relations (3.*i*). Under these assumptions, consider again the relation (6): $p(n') = A(n', n')^{-1}b(n', n')w$. Take the bundle of goods $d \in \Re^2_{++}$ as the standard of value, hence $d \cdot p = 1$, for every price vector p. Pre-multiplying relation (6) by d, one obtains:

$$d \cdot p(n') = 1 = d \cdot A(n', n')^{-1}b(n', n')w \qquad (7)$$

As we know, the elements of matrix $A(n', n')^{-1}$ and of vector $b(n', n')$ are increasing functions of n'. Thus, from (7) it follows that there exists a differentiable function n' of w such that:

$$\frac{dn'(w)}{dw} < 0 \qquad (8)$$

Moreover, if $n' < n$, at the constant wage rate w the rate of unemployment will increase over time. In this circumstance it is reasonable to assume that the wage rate decreases over time. Similarly, if $n' < n$, we assume that the wage rate increases over time. Thus, for the sake of simplicity, in what follows we assume that:

$$\frac{dw(t)}{dt} = (n'-n) \qquad (9)$$

By taking into account (8) and (9), we obtain:

$$\frac{dn'(w(t))}{dt} = \frac{dn'(w)}{dw}(n'-n) ,$$

from which it follows that:

$$\frac{dn'(w(t))}{dt} \overset{>}{\underset{<}{=}} 0 \quad \text{as} \quad n' \overset{<}{\underset{>}{=}} n .$$

The latter relations mean that if there is increasing (decreasing) unemployment, via adjustments in the wage rate, the uniform rate of growth n' increases (decreases) until the full employment rate of growth is reached. When the uniform growth rate of the economy attains the full employment rate, then the wage rate stops to change and so the uniform rate of growth of the economy. While this

case could resemble Kaldor's treatment of the price adjustment towards the full employment steady state, in our case the labour market adjustment works as a coordinating device since a change in the wager rate yields non cooperative adjustment of prices towards the "right" price vector $p(n)$.

5. Conclusions

In this paper a post-keynesian growth model with endogenous determination of mark up has been developed. The mark up is determined according to the view that firms want to maximize their growth rate under the condition that entry is barred, and that retained profits are the only source of accumulation. This approach allows to link the post-keynesian growth theory with the post-keynesian price theory on the one hand, and on the other hand the post-keynesian growth theory with the classical theory of production prices.

We have shown that within our context, the economy can be stuck into an underemployment or overemployment stationary state and that there is no automatic mechanism driving the economy towards a stationary state with full employment, even if nominal prices and real wage are perfectly flexible. It has been pointed out that the reason for this result is a coordination failure by firms in setting prices. If the wage rate is sticky, a coordination device could be a government intervention ensuring an "optimal" degree of competitiveness of the economy. By contrast, if the nominal wage is flexible, then a full employment stationary equilibrium will be attained as the labour market works as a coordination mechanism. This result makes our analysis close to the new keynesian macroeconomics.

Mathematical Appendix

From relation (3.i):

$$n_i(p_1, p_2; w) = \frac{s_c p_i y_i}{p_i k_{ii} + p_j k_{ij} + w} - s_c.$$

Fact A. For every $p_1, p_2, w \geq 0$:

$$n_i(0, p_j; w) = -s_c < 0;$$

$$n_i(p_i, 0; w) = \frac{s_c p_i y_i}{p_i k_{ii} + w} - s_c > -s_c;$$

$$\frac{\partial n_i(p_i, p_j; w)}{\partial p_i} = \frac{s_c y_i (p_j k_{ij} + w)}{(p_i k_{ii} + p_j k_{ij} + w)^2} > 0;$$

$$\frac{\partial n_i(p_i, p_j; w)}{\partial p_j} = \frac{-s_c k_{ij} p_i y_i}{(p_i k_{ii} + p_j k_{ij} + w)^2} < 0;$$

$$\frac{\partial^2 n_i(p_i, p_j; w)}{\partial p_i^2} = \frac{-2 s_c y_i k_{ii}(p_j k_{ij} + w)}{(p_i k_{ii} + p_j k_{ij} + w)^3} < 0;$$

$$\frac{\partial^2 n_i(p_i, p_j; w)}{\partial p_j^2} = \frac{2 s_c y_i k_{ij}^2 p_i}{(p_i k_{ii} + p_j k_{ij} + w)^3} > 0;$$

$\lim_{p_i \to \infty} n_i(p_i, p_j; w) = s_c(\frac{y_i}{k_{ii}}) > 0, \lim_{p_j \to \infty} n_i(p_i, p_j; w) = -s_c < 0$, where the first inequality follows

from the assumption on technology.

Fact B.

Price $p_i{}^*(p_j; w)$ is the solution to the following equation:

$$p_i^2 y_i k_{ji} + p_i(y_i(p_j k_{jj} + w) - k_{ii} p_j y_j) - p_j y_j(p_j k_{ij} + w) = 0.$$

The previous equation has one positive solution because $y_i k_{ji} > 0$ and $p_j y_j(p_j k_{ij} + w) > 0$.

Fact C.

$$\frac{\partial \Phi(p_i, p_j; w)}{\partial p_i} = \frac{\partial n_i(p_i, p_j; w)}{\partial p_i} - \frac{\partial n_j(p_i, p_j; w)}{\partial p_i} > 0;$$

$$\frac{\partial \Phi(p_i, p_j; w)}{\partial p_j} = \frac{\partial n_i(p_i, p_j; w)}{\partial p_j} - \frac{\partial n_j(p_i, p_j; w)}{\partial p_j} < 0.$$

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