Unemployment and Growth:

a Critical Survey

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Abstract - We present a survey of the literature on growth which allows for unemployment as a possible consequence of growth. It is not easy to find a systematic treatment of this topic as growth models have been elaborated for the major part under the assumption of full-employment. Notwithstanding full-employment cannot be considered the normal condition of both an advanced and developing countries. In this work we present several approaches to this topic, that are drawn by very different theoretical school of thought. We start from Harrod-Domar. Then we consider the Neoclassical approach to this problem: in the Solow (1956) model unemployment is completely ruled out; in the 'search' model, elaborated in the Nineties by Pissarides (1990) and (2000) unemployment arises in consequence of frictions occurring in the labour market; in neo-Schumpeterian growth theories, put foreword by Aghion and Howitt (1994), unemployment arises in consequence of innovations. Alongside these mainstream contributions we consider also two non-orthodox explanations of unemployment within a growing economy: those descending from the Goodwin (1967) and the Akerlof and Stiglitz (1969) models, that yield a cyclical behaviour of the unemployment rate and the wage share due to the conflictual aspects which characterize capitalistic economies, and that based on the analysis of structural change, elaborated by Pasinetti (1965), that emphasizes the possible arising of unemployment due to long-run lack of effective demand. All works are presented critically, and at the end of conclusion a prospect of future developments of research on the field is outlined.

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1 Introduction

In the major part of theoretical analysis of economic growth, full-employment is assumed as a normal long-run condition characterizing a growing economy. We find this element within recent Endogenous Growth Theory as well as in traditional Neoclassical Growth Theory originated by Solow (1956) and Swan (1956). Even some post-Keynesian growth frameworks consider full-employment as a configuration that the system tends to assume in the long-run.[1]

Notwithstanding full employment cannot be considered as the normal condition of both advanced and developing countries. Outside the period of the 'Great Crisis' of the Thirties, in the majority of advanced countries, the unemployment rate oscillates within a rather narrow band.

	1920		1933		1959-67		1982-92		1994-98	
Country	u	$g^\natural$	u	$g^\natural$	avg. u	g	avg. u	g	avg. u	g
Belgium	5.01	11.04	10.06	0.02	2.04	4.02	11.03	1.09	9.07	0.09
Denmark	3.00	4.08	14.05	0.06	1.04	3.06	9.01	1.01	11.9†	1.00
France	2.07	10.08	4.5‡	-0.9	0.07	4.03	9.05	1.06	12.01	0.07
Germany§	1.02	3.01	14.08	3.06	1.02	na	7.04	2.04	9.00	0.06
Ireland	na	na	na	0.07	4.06	3.08	15.5	3.07	11.02	3.06
Italy	1.07	-6.1	5.09	2.03	6.02	4.05	10.09	2.00	11.09	0.08
Netherlands	1.08	8.02	9.07	-1.3	0.09	3.06	9.08	2.01	5.09	1.03
Spain	na	2.07	na	0.07	2.03	6.01	19.00	2.08	21.08	1.03
UK	1.09	-5.6	13.09	3.01	1.08	2.03	9.07	2.01	8.00	1.02
Austria	2.05	3.00	16.03	-3.0	1.07	4.03	3.05	2.02	5.3†	0.09
Finland	1.01	10.07	6.02	4.06	1.07	4.04	4.08	0.05	14.02	1.09
Norway	5.06	5.01	9.07	3.05	2.01	3.05	3.02	2.02	4.06	1.07
Sweden	1.03	3.05	7.03	2.09	1.03	3.04	2.03	0.06	9.02	1.02
Switzerland	0.04	5.00	3.05	-0.1	0.02	3.00	0.07	0.09	5.0†	0.04
USA	3.09	-0.5	24.07.00	-1.0	5.03	2.07	7.01	1.08	5.03	0.08
Canada	na	-3.9	19.03	-0.3	4.09	na	9.06	0.09	9.04	1.00
Japan	na	2.04	na	3.06	1.05	8.06	2.05	3.03	3.04	0.05
Australia	4.06	1.06	17.04	3.05	2.02	2.09	7.08	1.06	8.06	3.02

Table 1: Unemployment rate (u) and growth rate of GDP per capita at constant prices (g), various countries, 1920-98.

Notes: † 1993 only; ‡ 1936; § The Federal Republic for the period 1959-91; ♮ Calculated using the period that includes the previous and the following year; na = not available.
Sources: Unemployment rate: Maddison *Monitoring the World Economy*, Oecd, Paris, 1995; Maddison *The World Economy – A Millenial perspective*, Oecd, Paris, 2001; *OECD Employment Outlook*, Various issues. Growth rate: Maddison *Monitoring the World Economy OECD* Oecd, Paris, 1995; World Bank, various years; World Development Indicators, Washington.

A curious aspect is the fact that the main corpus of 'modern' theories of growth has

[1] See Kaldor (1961). This interpretation does not concern all post-Keynesian models; see, for example, Pasinetti (1974, IV Essay, pp. 100-101).

been based on two contributions, by Solow (1956) and by Swan (1956), whose specific purpose was to solve the instability problem arisen in the model proposed by Harrod and Domar. We could say that Solow and Swan solved the problem so well that the model is 'practically bullet-proof' and refractory to the treatment of unemployment. It is perhaps for this reason that the first work that studied unemployment within a neoclassical growing economy has come out only after more than thirty years the works of Solow and Swan. The main works on this topic are Pissarides (1990), Aghion and Howitt (1994) and Pissarides (2000). Pissarides explains the permanence of unemployment in a steady growth as a consequence of the frictions due to the search process on the labour market. He presents this argument by developing seminal ideas proposed by Alchian (1969), Phelps (1968) Mortensen (1970a) and Mortensen (1970b). The main result pointed out by Pissarides within a growing economy is an *inverse relationship between growth rate and the unemployment rate*: on the basis of the assumption that firms' hiring costs increase at the same rate of productivity, a faster technological progress makes convenient for firms to anticipate future hirings, reducing thus the unemployment rate. In other terms an increase of the rate of growth, by lowering the actual rate at which future income will be discounted, increases today job creation, reducing thus the rate of unemployment. For this reason this phenomenon will be called later 'capitalization effect' by Aghion and Howitt, which criticized it as a too partial and incomplete explanation of the relationship between growth and unemployment, especially if referred to long-run. Aghion and Howitt (1994) drew the attention upon a more substantial phenomenon connected with growth: the fact that when productivity growth takes place through the introduction of new technologies, the consequent 'labor re-allocation' causes job-destruction and an increase of the unemployment rate. Their emphasis is thus placed on what they call 'creative destruction effect', that implies a *direct relationship between growth and unemployment*.

In both search models and neo-Schumpeterian models unemployment is treated as an *equilibrium* phenomenon, i.e. as the result of optimal choices of rational agents to various forms of rigidities; this permits to analyze with great detail the complex interrelations among the forces acting in favour or against it, and in some cases it permits to envisage the measures that can be undertaken to reduce or to avoid it. On the other side this approach, being too worried to bring back agents' behaviour to a micro-economic framework, runs the risk to overestimate some particular aspect of the problem and to loose, at the same time, some macro-aspects of the relationship between growth and distribution. In this respect the 'equilibrium' approach to unemployment may constitute rather a strait-jacket for the analysis than a way to make it rigorous.

By getting rid of this literature on 'equilibrium' approach to unemployment we can find a few interesting contributions that shed light on some other relevant aspects of the links between unemployment and growth. A first group of works is based on two seminal contributions appeared during the second half of the Sixties, conceived independently by Goodwin (1967) and by Akerlof and Stiglitz (1969). They focused their attention upon those forces that in a capitalistic growth process give rise to a *cyclical* evolution of the economy; in particular they showed how the long-run dynamics of a growing capitalistic economy may display persistent or dampened oscillations of the unemployment rate and of the share of wages around their full-employment equilibrium configuration. The cyclical behaviour of the economy lies in the description of the interactions of capitalists and workers that is described by a predator-prey model *à la* Lotka-Volterra model.[2]

Another independent attempt to analyze the effect of growth on unemployment may be found in the analysis of 'structural change' put forward by Pasinetti (1965), (1981) and (1993). Within this framework unemployment arises as the 'normal' consequence of a growth process, as the result of interaction between technological change and the saturation of individual demand. The demand side of the problem comes thus to play its relevant part, in connection with technological change, in an analysis concerning long-run unemployment.

The sharpness with which these results come out and the intelligibility of the reasons which engender them, made these non-orthodox approaches very fruitful and appreciated. In this survey the peculiarities of the different approaches will be emphasized and discussed. Notwithstanding there is nothing to prevent to consider both orthodox and non-orthodox contributions side by side, either to shed more light on the topics analyzed, or to question each approach about its greatest strengths and its weakness.[3]

The survey here presented aims to evaluate critically all these approaches, in order to provide a comprehensive outline of theoretical explanation of the links between unemployment and growth.

This is the structure of the work. In section 2 we will present the Harrod-Domar model, and we will recall the kind of instability that emerges in a growing economy according to this model. In section 3 we focus upon the Neoclassical visions of the links between growth and unemployment: in subsection 3.1 we will review briefly the solution proposed by Solow to the Harrodian instability; in subsection 3.2 we will present the the basic elements of 'search' models and we will see them in connection with a growing economy; in subsection 3.3 we will recall the neo-Schumpeterian analysis of growth and we will see its consequences for employment. In section 4 we will present two heterodox approaches to our topic: the growth-cycle models *à la* Goodwin and Akerlof-Stiglitz (subsection 4.1) and the structural change analysis, put foreword by Pasinetti (subsection 4.2). Section 5 concludes.

2 The Harrod-Domar model

We can fix the beginning of 'modern' growth theory with the Harrod-Domar model and the debate that arose around the instability of its solution. Let us recall here the emergence of instability in the Harrod-Domar model. Consider an economic system represented in aggregate terms. The level of investments, I, has a twofold role: i) they determine the

[2]More than one hundred and thirty works have been published as extensions and generalizations of the Goodwin model. Inexplicably less attention has aroused the Akerlof-Stiglitz model.

[3]It seems surprising that in a recent survey written on the same topic (Aricò (2003)) there is no trace of any non-mainstream contribution; the oldest reference explicitly linked with growth and unemployment quoted in that survey is dated 1986.

global amount of effective demand, Q, i.e.

$$\frac{1}{s} \cdot I = Q, \tag{1}$$

and ii) they increase the total productive capacity of the economy, P, i.e.

$$\frac{1}{v} \cdot I = \frac{\mathrm{d}P(t)}{\mathrm{d}t}, \tag{2}$$

where s is the marginal propensity to save out of income and v is the ratio between capital (K) and output (Q), i.e. $v = K/Q$. Parameters s and v are determined independently each other, the former reflects individuals habits concerning consumption and saving and the latter is determined by technology. As known both s and v are assumed to be given outside the model. Hence there are no reasons for which the effects i) and ii), described by equations (1) and (2), are compatible with the preservation of full employment of productive capacity. According to Domar this goal is attained in the long-run i) if this condition is attained at the beginning or our period:

$$P(0) = Q(0), \tag{3}$$

and ii) if we have a uniform expansion of P and Q:

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = \frac{\mathrm{d}Q(t)}{\mathrm{d}t}. \tag{4}$$

To solve the model, derive (1) with respect to t, substitute this derivative and equation (2) in (4) and obtain:

$$\frac{1}{I} \cdot \frac{\mathrm{d}I}{\mathrm{d}t} = \frac{s}{v} \quad \text{or, by integration,} \quad I(t) = I(0)e^{g_w t}, \quad \text{where} \quad g_w := s/v. \tag{5}$$

g_w is the equilibrium rate of growth of investment, but also of income, $Q(t) = (1/s)I(t) = Q(0)e^{g_w t}$, of consumption, $C(t) = (1-s)Q(t) = C(0)e^{g_w t}$ and of capital, $K(t) = vQ(t) = K(0)e^{g_w t}$. This rate has been called 'warrented' rate of growth, as it is the rate at which investments, income, consumption and capital *have to increase* in order to guarantee the full employment of productive capacity.

Up till now we have been considering full employment of productive capacity but we have said anything about full employment of labour force. There are no reason for which the former implies the latter. To this purpose it is necessary to introduce an additional condition. Suppose that the labour force, N, increases at a constant rate, n, and that, thanks to technical progress, the average productivity of each worker increases at a constant rate, ρ, exogenous with respect to the model. Thus labour force, measured in terms of efficiency units, L, increases at a constant rate, $n + \rho$,

$$L(t) = N(t)e^{\rho t} = L(0)e^{(n+\rho)t}. \tag{6}$$

By taking into account these two phenomena the total product will vary as follows:

$$Q(t) = q(t)L(t) = q(0)L(0)e^{(n+\rho)t},$$

where $q := Q/L$ is the product per efficiency unit. $g_n := n + \rho$ has been called by Harrod, 'natural' rate of growth, as it indicates the maximum rate of growth allowed by the demographic trend and the evolution of technology. From this it follows that the condition to fulfil at the same time the full employment of productive capacity and of the labour force is:

$$g_w = g_n, \quad \text{or, equivalently,} \quad \frac{s}{v} = n + \rho. \tag{7}$$

(7) is the Harrod-Domar equation. It individuates the rate of growth of investment, income, consumption, capital, at which is preserved, at the same time, full employment of both productive capacity and labour force is preserved along time. Such a path is called 'balanced growth path', or 'long-run equilibrium growth path'. As the four parameters involved in (7) are exogenous to the Harrod-Domar model, equality (7) may happen to be verified only by a fluke.[4] Hence the economy cannot evolve, in general, along a long-run equilibrium growth path. It may happen that $s/v < n + \rho$ or $s/v > n + \rho$: in the former case there will be an increasing *unemployment* of labour force; in the latter case there will be an increasing excess demand of labour and the emerging of inflationary pressures. The Harrod-Domar long run equilibrium growth path is thus instable: if the system happens to be exactly on it, it will remain there forever; otherwise it moves away from this path indefinitely. It is in relation to this 'knife-edge' instability problem that the Solow's contribution becomes relevant.

3 Neoclassical approaches

3.1 The Solow's solution to Harrod-Domar instability

The source of instability of the Harrod-Domar growth path is the assumption that parameters s, v, n, and ρ are given exogenously and independently one another. This is a rather extreme assumption. Some of them may be affected by the disequilibrium situation and change in response to this disequilibrium.

Solow inserted the Harrod-Domar model within a neoclassical framework to study how can vary the capital/output ratio in response to a disequilibrium situation. According to standard neoclassical theory of production v is a technological parameter whose level is determined by profit-maximizing choices of firms. These choice process is described at aggregate level as follows. National income is supposed to be produced with different (infinite) production techniques summarized by a well behaved aggregate production

[4]This extreme vision is typical of the textbook presentations of the Harrod-Domar model. Actually Harrod allows for a certain (small) of substitutability between factors; but he did not explicitly considered it as his analysis is carried out with a *constant* interest rate. For further details see Commendatore, D'Acunto, Panico, and Pinto (2003, p.106 and, specially, footnotes 9 and 10).

function,[5]

$$Q = F(K, L).$$

In this way equation from equation (1) we obtain that capital evolves according to equation:[6]

$$I \equiv \dot{K} = sF(K, L). \tag{8}$$

Let

$$q := Q/L, \quad k := K/L \quad \text{and, therefore,} \quad v = K/Q, \tag{9}$$

where q and k are the output/labour and the capital/labour ratios. Thanks to homogeneity properties of F (assumption 3, footnote 5) we may re-express equation (8) in terms of k only:

$$\dot{k} = sf(k) - (n + \rho)k, \tag{8$'$}$$

where $f(k) := F(K/L, 1)$ is the production function in intensive form, and express the total product per each labour efficiency unit. Thanks to assumptions 2 and 3 of footnote 5, $f(k)$ is an increasing, strictly concave, continuous and differentiable function of k; strict concavity entails that

$$kf'(k) < f(k). \tag{10}$$

(8′) is a non-linear first order differential equation in k; its steady state equilibrium value is given by $\dot{k} = 0$, i.e.

$$k = k^* \quad \text{such that} \quad sf(k^*) = (n + \rho)k^*. \tag{11}$$

Steady state k^* is locally asymptotically stable: in fact

$$\left.\frac{\mathrm{d}\dot{k}}{\mathrm{d}k}\right|_{k=k^*} = sf'(k^*) - (n + \rho) < 0$$

thanks to (10) and (11).

We can now return to the Harrod-Domar instability problem: observe, at first, that, thanks to (11) and (9), if $k = k^*$ then

$$s/v^* = n + \rho \equiv g_w, \tag{12}$$

[5] $F(K, L)$ is a 'well-behaved' production function if the following conditions hold:

1. for $K \geq 0$ and $L \geq 0$, $F(K, L)$, $\frac{\partial F}{\partial K}$, $\frac{\partial F}{\partial L}$, $\frac{\partial^2 F}{\partial K^2}$, $\frac{\partial^2 F}{\partial L^2}$ are well-defined and continuous; moreover $F(0, 0) = F(K, 0) = F(0, L) = 0$ e $F(K, L) \geq 0$ for $K \geq 0$, $L \geq 0$ and $\lim_{K \to 0} \frac{\partial F}{\partial K} = \lim_{L \to 0} \frac{\partial F}{\partial L} = +\infty$ and $\lim_{K \to +\infty} \frac{\partial F}{\partial K} = \lim_{L + \infty} \frac{\partial F}{\partial L} = 0$;
2. $\dfrac{\partial F}{\partial K} \geq 0$, $\dfrac{\partial F}{\partial L} \geq 0$; $\dfrac{\partial^2 F}{\partial K^2} \leq 0$ and $\dfrac{\partial^2 F}{\partial L^2} \leq 0$ (decreasing marginal products);
3. $F(\lambda K, \lambda L) = \lambda F(K, L)$, $\forall \lambda > 0$ (constant returns to scale).

[6] Throughout the work symbol $\dot{x}$ will mean the derivative with respect to time of variable x, that is, $\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t}$.

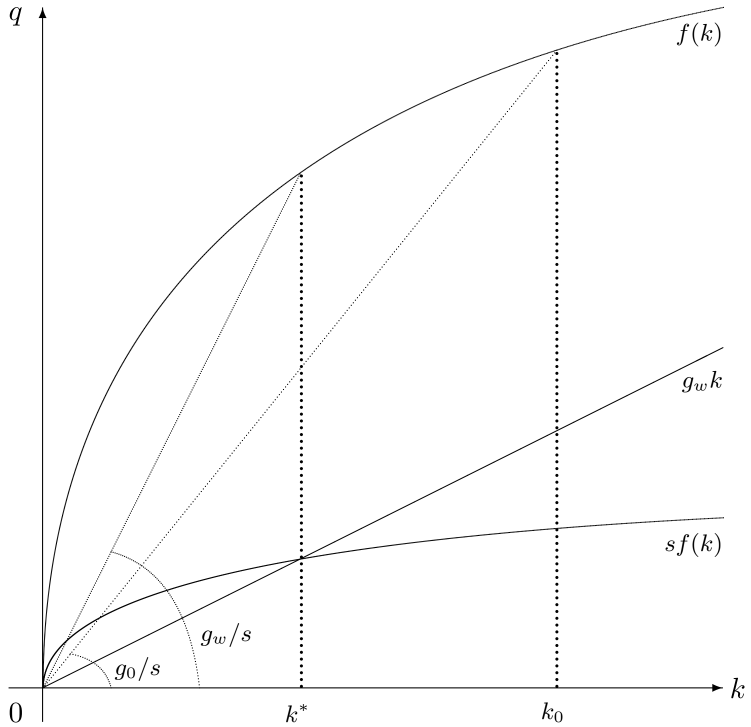

Figure 1: Adjustment of g_w to g_n in the Solow model

where $v^* = f(k^*)/k^*$; i.e. the steady state equilibrium of the Solow model coincides with the long-run equilibrium of the Harrod-Domar model. The asymptotical stability of k^* means thus that if we are not in the Harrod-Domar long-run equilibrium path, we move towards it asymptotically. More precisely consider the case in which at a given time $t = t_0$ condition (7) is not satisfied; to fix ideas suppose that

$$s/v_0 < n + \rho. \tag{13}$$

This situation would correspond to a growth path in which we will observe *increasing unemployment*. But in this context the capital-output ratio is a variable, $v = k/f(k)$; hence (13) means that we have an initial capital-labour ratio, k_0, higher than the steady state level; in fact if 13 holds, then $sf(k_0) < (n + \rho)k_0$ and from (8′) we observe that k will move in the 'right' direction, i.e. it will decrease up to k^*, in correspondence of which, thanks to (11), (12) holds. (Reverse the analysis if $s/v_0 > n + \rho$.) All this can be seen in figure 1. Hence within the Solow version of the Harrod-Domar model *the full-employment long-run equilibrium growth path is asymptotically stable.*

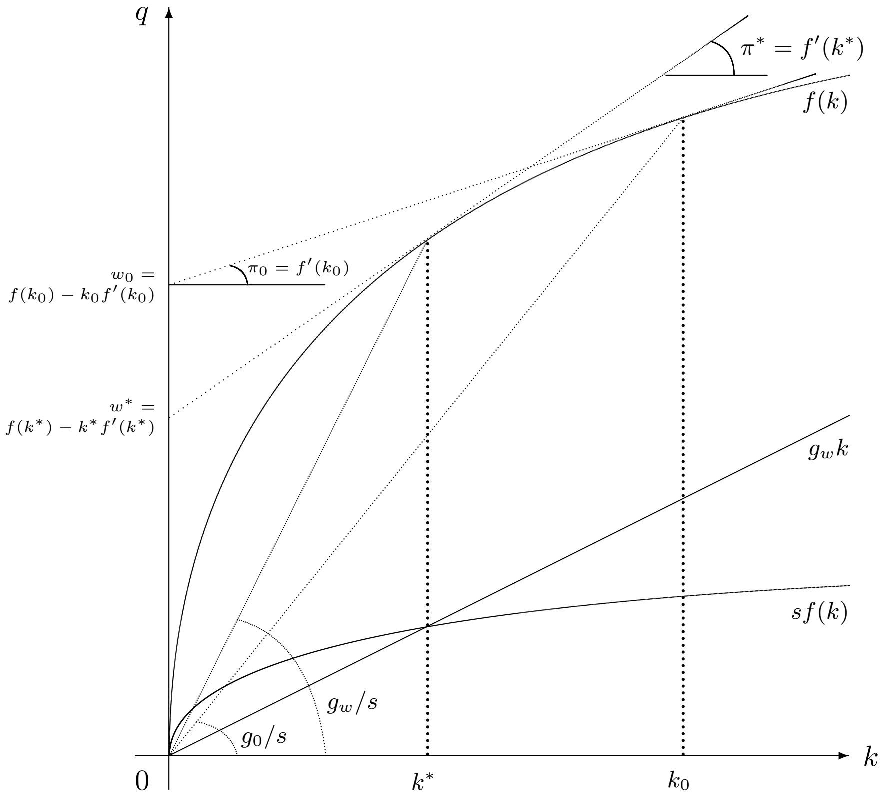

Figure 2: Adjustment of g_w to g_n in the Solow model

It is possible going further, and describe how factors markets 'support' this adjustment process to the Harrod-Domar equilibrium path. In a neoclassical world, at given prices, profit maximizing firms will demand capital and labour till when their marginal productivities equal their prices, both expressed in terms of the product, Π/P and W/P, i.e.

$$\frac{\partial F}{\partial K} \equiv f'(k) = \frac{\Pi}{P} =: \pi \tag{14a}$$

$$\frac{\partial F}{\partial L} \equiv f(k) - kf'(k) = \frac{W}{P} =: w. \tag{14b}$$

It can be observed in Figure 2 that the transition from v_0 to v^* is supported by a *decrease* of the wage rate from w_0 to w^* and an *increase* of the rate of profit from π_0 to π^*.

It should be noted that in this economy there is *never* unemployment, even when the economy is out of its long-run equilibrium growth path: in that case the system would give rise to unemployment *in the future*, if there were not the adjustment of capital as described, but at each instant relative factor prices w and π vary in order to adapt capital and labour demand (implicitly given by equations (14)) to their respective supply, given by the solution for $K(t)$ of (8) and by (6). Hence both factor markets clear at each date thanks to flexibility of prices, and the system moves towards a full-employment growth path, in correspondence of which relative prices will be constant.

3.2 Equilibrium Unemployment Theory: the 'search' model

From section 3.1 it is evident how the Solow model constitutes a radical departure from the Harrod-Domar model, rather than a generalization, at least from the point of view here adopted: from a situation where the disequilibrium (unemployment or inflationary pressures) was the rule, we have been shifted to a situation where full-employment and equilibrium is attained *both* in the transition towards the long-run equilibrium *and* in correspondence of it. It is really another economy.

This setting has pervaded the main part of growth analysis since the Sixties. It is thus not surprising that it has turned out to be very difficult to find theoretical studies of unemployment within a growth context. An important attempt in this direction has been the extension of the notion of the 'natural' rate of unemployment to a growing economy. This attempt was carried out by Pissarides (1990) and (2000) in his *Equilibrium Unemployment Theory.* In this contribution unemployment origins from the frictions that take place in the labour market in the matching process between unemployed workers and firms with vacant jobs. The description of this matching process is the starting point of this model.

3.2.1 Matching and separation

Trade in the labour market is an economic activity. It is uncoordinated, time-consuming and costly for both firms and workers. We describe this activity through a well-behaved *matching function.* Let L the number of workers belonging to the labour force; the matching function express the number of jobs formed at a given moment of time, mL, as a function of the workers looking for a job, uL, and the number of workers each firm is looking for to fill a job slot, vL:

$$mL = m(uL, vL), \tag{15}$$

where u and v are, respectively, the unemployment rate and the vacancy rate (the ratio of vacant jobs over the labour force). $m(\cdot,\cdot)$ is supposed to be a continuously differentiable function. Moreover it is increasing with respect to both arguments, ($m_1 > 0$ and $m_2 > 0$), concave (hence $m_{11} < 0$ and $m_{22} < 0$), homogeneous of degree 1 and such that $m(0, vL) = m(uL, 0) = m(0, 0) = 0$. Thanks to the homogeneity properties we can write $m(\cdot,\cdot)$ as a function of the ratio between the vacancy rate the unemployment rate only:

$$\frac{mL}{vL} = m\left(\frac{u}{v}, 1\right) =: \mu(\tau) = \mu(\theta^{-1}) =: q(\theta), \quad \text{where} \quad \theta := \frac{v}{u} = \frac{1}{\tau}. \tag{16}$$

Hence:

- $q(\theta) = \frac{\text{number of matchings}}{\text{number of vacant jobs}}$: rate at which a vacant job becomes filled;
- $1/q(\theta)$: mean duration of a vacant job;
- $\theta q(\theta) = \frac{\text{number of matchings}}{\text{number of unemployed workers}}$: rate at which an unemployed worker finds a job;
- $1/\theta q(\theta)$: mean duration of unemployment.

Functions $\mu(\tau)$ and $q(\theta)$ inherit some properties from $m(\cdot, \cdot)$. In particular from $m(uL, vL) \equiv vL \cdot \mu\left(\frac{uL}{vL}\right)$ we obtain

$$\mu'(\tau) = m_1 > 0 \quad \text{and} \quad \mu''(\tau) = vLm_{11} < 0. \tag{17}$$

On the other side as $q(\theta) = \mu(\theta^{-1})$ we obtain:

$$q'(\theta) = -\mu'(\theta^{-1}) \cdot \theta^{-2} \quad \text{and} \quad q''(\theta) = \mu''(\cdot) \cdot \theta^{-4} + 2\mu'(\cdot) \cdot \theta^{-3}. \tag{18}$$

From (17)

$$q'(\theta) < 0, \tag{19}$$

while the sign of $q''(\theta)$ is not defined. Moreover from (17) μ is concave and $\mu(0) = 0$, hence $\mu(\tau) \leq \tau\mu'(\tau)$; re-writing this in terms of q we obtain the following interval for the elasticity of q with respect to θ, $\eta(\theta)$:

$$-1 \leq \eta(\theta) := \frac{q'(\theta) \cdot \theta}{q(\theta)} \leq 0. \tag{20}$$

3.2.2 Firms

Each firm i is characterized by the same well-behaved production function, $F(K_i, pE_i)$ where K_i and E_i are the capital and employment (measured as the number of workers) of firm i and p is a labour-augmenting productivity parameter (thus pE_i is the employment of firm i measured in efficiency units). The firm buys new capital good, $\dot{K}_i$, at the price of his output and pays to each worker a real wage rate, w, taken as given by the firm (we will see later how w is determined). There are no costs of adjustment for capital. There are cost of adjustment for employment, that are supposed linear with respect to the number V_i of vacancies open by firm i; on the other side it is supposed that the cost of each vacancy is proportional to productivity, on the ground that it is more costly to hire more productive workers; thus if pc is the cost of each vacancy the cost of adjustment for employment are pcV_i. The present-discounted value of the firm's expected profits is thus:

$$\int_0^{+\infty} e^{-rt}[F(K_i, pE_i) - \dot{K}_i - wE_i - pcV_i - \delta K_i]\mathrm{d}t, \tag{21}$$

where δ is the depreciation of the capital stock and r is the interest rate. In this exposition of the model we suppose that r is given outside of the model.[7] E_i is not under the complete

[7] Pissarides treats also the case of an endogenous interest rate (see Pissarides (2000, § 3.4)).

control of the firm. This latter can affect E_t through the number of open vacancies, yielding workers at the rate $q(\theta)$; on the other side a job-worker pair may separate as a consequence of a structural shift of demand. Let s the probability at which an employed worker falls into unemployment due to such a shift. Hence the firm's labour force changes according to

$$\dot{E}_i = q(\theta)V_i - sE_i. \tag{22}$$

Firm i maximizes (21) with respect to K_i and E_i subject to (22). Substituting the constraint into the objective function we obtain the following problem of calculus of variation:

$$\max_{K_i,E_i} \Pi_i(K_i, E_i, \dot{K}_i, \dot{E}_i) :=$$

$$= \int_0^{+\infty} e^{-rt} \left(F(K_i, pE_i) - \dot{K}_i - wE_i - pc\frac{\dot{E}_i + sE_i}{q(\theta)} - \delta K_i \right) \mathrm{d}t. \tag{23}$$

The optimal sequence of K_i and E_i satisfies the following Euler conditions:[8]

$$\Pi_{iK_i} = \dot{\Pi}_{i\dot{K}_i} \quad : \quad e^{-rt}\left[F_1(K_i, pE_i) - \delta\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[e^{-rt}(-1)\right]$$

$$\Pi_{iE_i} = \dot{\Pi}_{i\dot{E}_i} \quad : \quad e^{-rt}\left[p\ F_2(K_i, pE_i) - w - pc\frac{s}{q(\theta)}\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[e^{-rt}\left(\frac{-pc}{q(\theta)}\right)\right],$$

that is,

$$F_1(K_i, pE_i) = r + \delta, \tag{24a}$$

$$p\ F_2(K_i, pE_i) = w + \frac{(s+r)pc}{q(\theta)}. \tag{24b}$$

Thanks to homogeneity properties of the production function we can re-express marginal productivities of equations (24) as functions of the ratio $K_i/pE_i = k_i$ only; moreover by observing that in (24) all variables other than k_i are not indexed by i, as they are 'market' variables, we can conclude that each firm chose the same capital-labour ratio, i.e. $k_i = k$; thus we can re-write conditions (24) as follows:

$$f'(k) = r + \delta, \tag{24a$'$}$$

$$p[f(k) - kf'(k)] = w + \frac{s+r}{q(\theta)}pc. \tag{24b$'$}$$

Equation (24a′) is the usual marginal productivity condition for the demand of capital (MPK); when (24a′) is satisfied, (24b′) is the the analogous of the usual marginal condition for the demand of labour; it differs from equation (14b) for the term $(s+r)pc/q(\theta)$, which corresponds to the expected capitalized value of the firm's hiring cost. It has been called Job Creation condition (JC). If $c = 0$ (24b′) would coincide with (14b); it represents thus,

[8] See, for example, Kamien and Schwartz (1981, p. 105).

in implicit terms, the labour demand function or, from another point of view, the supply of jobs.

Firms vacancies in steady state are obtained by setting $\dot{E}_i = 0$ in (22):

$$V_i/E_i = s/q(\theta). \tag{25}$$

Condition (25) entails that in the steady state all firms chose the same ratio of vacancies to employment, θ, and therefore (25) also gives the ratio of all vacancies to total employment. As $\sum_i V_i = \theta u L$ and $\sum_i E_i = (1-u)L$, then (25) becomes

$$u = \frac{s}{s + \theta q(\theta)}. \tag{26}$$

(26) is known as Beveridge curve (BC). It can be proved that BC is *decreasing* and *convex*.[9]

3.2.3 Wages

Up till now we have a model with three equations, (24a′), (24b′) and (26), in four unknowns, k, w, u and θ. We need an additional equation. We can find it in the forces that determine the real wage rate.

This equation is obtained by observing that in equilibrium occupied jobs must yield a total return that is strictly greater than the sum of the expected returns of a searching firm and a searching worker. If the firm and the worker who are together separate, each will have to go through an expansive process of search before meeting another partner. Hence a realized job match yields some pure economic rent, equal to the sum of the expected search costs for the firm and for the worker (including foregone wages and profits). We assume that the monopoly rent is shared according to the Nash solution to a bargaining problem. The wage rate for a job is fixed by the firm and the worker after they meet. Because all jobs are equally productive and all workers place the same value on leisure, we will find that the wage rate fixed for each job is the same everywhere. Each firm and

[9] In fact,

$$\frac{\mathrm{d}u}{\mathrm{d}\theta} = -s\frac{q(\theta) + \theta q'(\theta)}{[s + \theta q(\theta)]^2} < 0 \tag{27}$$

thanks to (20). Moreover

$$\begin{aligned} \frac{\mathrm{d}^2 u}{\mathrm{d}\theta^2} &= -s\left\{-2[s+\theta q(\theta)]^{-3} \cdot [q(\theta) + \theta q'(\theta)]^2 + [s+\theta q(\theta)]^{-2} \cdot [2q'(\theta) + \theta q''(\theta)]\right\} \\ &= 2s\frac{[q(\theta) + \theta q'(\theta)]^2}{[s+\theta q(\theta)]^3} - s\frac{2q'(\theta) + \theta q''(\theta)}{[s+\theta q(\theta)]^2}; \end{aligned} \tag{28}$$

the first addendum of (28) is positive; the second one, thanks to (18), can be written as:

$$2q'(\theta) + \theta q''(\theta) = \mu''(\cdot)\theta^{-3} < 0, \tag{29}$$

hence

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\theta^2} > 0. \tag{30}$$

each worker are too small to influence the market, so when they meet they fix their own wage rate by taking the behaviour in the rest of the market as given. Let w_i the wage rate negotiated by a given firm and by a given worker after their meeting. To understand how w_i will be determined we have to describe costs and benefits for each worker and for each firm.

Let Ω_i denote the present-discounted value of the expected income stream of an employed worker when the real wage rate is w_i and let U denote the present-discounted value of the expected income stream of an unemployed worker. An employed worker earns w_i and can lose his job becoming unemployed with probability s. Hence Ω_i, that can be considered the value of the asset 'employed worker's human capital', must satisfy:

$$\Omega_i = \frac{w_i - s(\Omega_i - U)}{r}. \tag{31a}$$

The unemployed worker earns a real return, ξ, which is measured in the same units as real wages. ξ may have different components. (It will be seen that the specification of ξ is very—perhaps too—important for the results of the model.) Moreover in unit time the worker expects to move into employment with probability $\theta q(\theta)$; hence U, the value of the asset 'unemployed worker's human capital', must satisfy:

$$U = \frac{\xi + \theta q(\theta)(\Omega_i - U)}{r}. \tag{31b}$$

Derive now an expression of firms' net return. When the job is vacant the firm is engaged in hiring at fixed cost pc per unit time. The firm's hiring activity returns a worker with probability $q(\theta)$. Thus the value of the asset value of 'vacant job' is given by

$$V = \frac{-pc + q(\theta)(J_i - V)}{r}, \tag{32a}$$

where J_i is the present discounted value of profits arising from that job. The job yields net return given by $p[f(k) - kf'(k)] - w_i$ to the firm; the job also runs a risk s of an adverse shock, which lead to the loss of J_i with probability s. J_i is thus determined by the asset-value condition:

$$J_i = \frac{p[f(k) - kf'(k)] - w_i - sJ_i}{r}. \tag{32b}$$

In equilibrium $V = 0$, therefore (32a) entails:

$$J_i = pc/q(\theta). \tag{33}$$

Remembering the meaning of $1/q(\theta)$, (33) states that in equilibrium the expected profit from a filled job is exactly equal to the expected cost of hiring a worker.

In the bargaining between a worker and a firm w_i will be fixed in order to maximize the weighted product of the worker's and the firm's net return from the job, $\Omega_i - U$ and

$J_i - V$:

$$\max_{\Omega_i, J_i} (\Omega_i - U)^\beta \cdot (J_i - V)^{1-\beta}, \quad 0 \leq \beta \leq 1, \tag{34a}$$

subject to the constraint:

$$J_i + \Omega_i - V - U = \bar{X}. \tag{34b}$$

From the first order condition of problem (34) we obtain:

$$\Omega_i - U = \beta(J_i + \Omega_i - V - U) = \frac{\beta}{1-\beta}(J_i - V). \tag{35}$$

By substituting the expressions of Ω_i and of J_i obtained respectively by (31a) and (32b) into (35) and by taking into account equation (24a′) and that $V = O$ we obtain the expression of of the wage rate:

$$w_i = \beta p[f(k) - (r + \delta)k] + (1 - \beta)rU. \tag{36}$$

From (36) we see that all jobs offer the same wage rate; hence as expected, $w_i = w$ and (35) may be written without index i; to know the expression of w it remains to know U; to this purpose take into account (33) and that that $V = 0$; (35) thus entails that $\Omega - U = \frac{\beta}{1-\beta}\frac{pc}{q(\theta)}$; by substituting this expression of $\Omega - U$ into (31b) we get that

$$rU = \xi + \frac{\beta}{1-\beta} pc\theta. \tag{37}$$

Thus (36) may be written as:

$$w = (1 - \beta)\xi + \beta p[f(k) - (r + \delta)k + c\theta], \tag{36′}$$

which is the fourth equation of the model; call it Wage Equation (WE). Now the model is determined: four equations,

$$(MPK) \qquad f'(k) = r + \delta, \tag{24a′}$$

$$(JC) \qquad p[f(k) - kf'(k)] = w + \frac{s + r}{q(\theta)} pc, \tag{24b′}$$

$$(BC) \qquad u = \frac{s}{s + \theta q(\theta)}, \tag{26}$$

$$(WE) \qquad w = (1 - \beta)\xi + \beta p[f(k) - (r + \delta)k + c\theta], \tag{36′}$$

in four unknowns, k, θ, u and w. (24a′) determines $k = k^*$; given k^* (24b′) determines $\theta = \theta^*$; given θ^* (26) determines $u = u^*$ and given k^* and θ^* (36′) determines $w = w^*$.

We have thus obtained a *steady-state equilibrium with unemployment* within a neoclassical model. Unemployment arises in consequences of frictions on the labour market concerning the matching process between unemployed workers and firms with vacant jobs.

We can obtain immediately a relevant comparative static result. Substitute (24a′) into (36′) and what thus obtained into (24b′); evaluate at equilibrium and obtain:

$$(1-\beta)p[f(k^*) - k^* f'(k^*)] = (1-\beta)\xi + \left(\beta\theta^* + \frac{s+r}{q(\theta^*)}\right) pc;$$

by differentiating this latter with respect to θ and p we obtain:

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}p} = \frac{(1-\beta)\xi}{p^2\{\beta c - (s+r)q'(\theta^*)/[q(\theta^*)]^2\}} > 0;$$

thus the following proposition holds:

PROPOSITION 1 *A labour-augmenting productivity shock increases market tightness and decreases, through* (26)*, the unemployment rate.*

3.2.4 Unemployment income

In order to draw the main conclusions on the way that growth affect unemployment Pissarides (2000, p. 70ff) introduces a dependence between the fixed actual imputed income during unemployment, ξ, and the productivity parameter, p. Look at this point with care because it is crucial in Pissarides analysis.

Unemployment income may consist of i) income received during unemployment (for instance a subsidy); ii) income earned doing odd jobs in a secondary sector of the economy; iii) the imputed value of time to unemployed workers. It is reasonable to assume that along the steady state: i) is fixed in terms of the prevailing wage rate, through some form of indexation; ii) is in fixed proportion to income from work in the primary sector; iii) measures the compensation that the worker requires in order to give up his time for work. This latter should be a function of his wealth; hence for this channel ξ should depend on both human and nonhuman wealth.

Hence from channels i) and ii) ξ should depend on w; as far as channel iii) is concerned we could observe that human wealth for unemployed workers is given by U, i.e. the 'asset value' of the worker during search. Denote by A nonhuman wealth. Thus we could write

$$\xi = \gamma w + \zeta r(A+U), \quad 0 < \gamma < 1, \quad 0 < \zeta < 1. \tag{38}$$

Pissarides adds:

> "although it may be reasonable to assume that nonhuman wealth is independent of market outcome in the short run, in the longer run it adapts to labor-market earnings.
>
> If wealth plays an important role in determining reservation wages and the bargaining stand of workers, the slow response of it to long-term changes in labor market condition could explain persistent effects of productivity changes on unemployment. [...] In the analysis that follows, however, we will concentrate on the longer-run steady state property of equilibrium, where nonhuman

> wealth, if it matters, has had time to adjust the labor market equilibrium. Under these circumstances there is no loss of generality if we ignore nonhuman wealth. Pissarides (2000, p. 73)

Hence we could set $A = 0$ and rewrite (38) as follows

$$\xi = \gamma w + \zeta r U, \quad 0 < \gamma < 1, \quad 0 < \zeta < 1. \tag{38$'$}$$

By substituting (38′) into (37) we yield:

$$rU = \frac{\gamma}{1-\zeta} w + \frac{\beta}{(1-\beta)(1-\zeta)} pc\theta. \tag{37$'$}$$

Re-substituting into (38′) we obtain:

$$\xi = \frac{\gamma}{1-\zeta} w + \frac{\beta\zeta}{(1-\beta)(1-\zeta)} pc\theta. \tag{38$''$}$$

Finally by inserting this expression of ξ into the wage equations we get:

$$w = \beta H \left\{ [f(k) - (r+\delta)k] + \frac{\zeta}{1-\zeta} \, c\theta \right\} p, \tag{36$''$}$$

where $H := (1-\zeta)/[(1-\zeta) - \gamma(1-\beta)] > 0$.

The key property of (36″) is that now the real wage is proportional to the productivity parameter, p; the proportionality factor depends positively on labour-market tightness, θ and the valuation of leisure, ζ. The first consequence of this can be seen by substituting (36″) and (24a′) into JC (equation (24b′)),

$$(1 - \beta H)[f(k) - kf'(k)] = \beta H \frac{\zeta}{1-\zeta} c\theta + \frac{s+r}{q(\theta)} c : \tag{39}$$

p cancels out on both side; thus its solution, i.e. labour-market tightness in steady state, is independent of the productivity parameter, p.

With these assumptions on unemployment income the equations of the model appear as follows:

$$(MPK) \qquad f'(k) = r + \delta, \tag{24a$'$}$$

$$(JC) \qquad p[f(k) - kf'(k)] = w + \frac{(s+r)pc}{q(\theta)}, \tag{24b$'$}$$

$$(BC) \qquad u = \frac{s}{s + \theta q(\theta)}, \tag{26}$$

$$(WE) \qquad w = \beta H \left\{ [f(k) - (r+\delta)k] + \frac{\zeta}{1-\zeta} c\theta \right\} p. \tag{36$''$}$$

As before they are four equations in our four unknowns, k, θ, u and w. The only difference with respect to model (24a′), (24b′), (26) and (36′) is equation (36″). But this difference alters significantly the causal structure of the system: (24a′) determines $k = k^*$; given k^* (24b′) determines $\theta = \theta^*$; given θ^* (26) determines $u = u^*$ and given k^* and θ^* (36″)determines $w = w^*$; hence thanks to it, the following result holds:

PROPOSITION 2 *Labour-augmenting productivity shocks do not affect unemployment; they are fully absorbed by wages, so that equilibrium unemployment does not respond to them.*

Compare this conclusion with that reached in Proposition 1: the proportionality of ξ with productivity has thus restored the usual result of an unmodifiable natural rate of unemployment.

3.2.5 Technological Progress: the 'Capitalization' Effect

Introduce now exogenous disembodied labour-augmenting technical progress. Suppose that

$$p(t) = p_0 e^{\rho t}, \quad p_0 > 0, \quad \rho < r. \tag{40}$$

As now parameter p is a function of time, Euler's conditions of problem (23) must be re-formulated as follows:

$$e^{-rt}\left[F_1(K_i, pE_i) - \delta\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[e^{-rt}(-1)\right]$$

$$e^{-rt}\left[p\ F_2(K_i, pE_i) - w - pc\frac{s}{q(\theta)}\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[e^{-rt}\left(\frac{-p_0 e^{\rho t} c}{q(\theta)}\right)\right];$$

that is

$$f'(k) = r + \delta, \tag{41a}$$

$$f(k) - kf'(k) = \frac{w}{p} + \frac{s + r - \rho}{q(\theta)}\ c. \tag{41b}$$

The model is now still constituted by four equations: the MPK condition, (41a), the JC condition (41b), the WE (36″), and the BC (26). The equilibrium solution is obtained as follows: (41a) determines $k = k^{\diamond}$, $k^{\diamond}$ and (39) determine $\theta = \theta^{\diamond}$; $k^{\diamond}$ and $\theta^{\diamond}$ determine $w = w^{\diamond}$ through (36″) and, finally, $\theta^{\diamond}$ determines $u = u^{\diamond}$ through (26).

By substituting (36″) into (41b) we obtain the equation that determines labour-market tightness:

$$(1 - \beta H)[f(k) - kf'(k)] = \beta H\frac{\zeta}{1-\zeta}c\theta + \frac{s + r - \rho}{q(\theta)}\ c, \tag{42}$$

which does not depend—as equation (36″)—by p, but depends on its growth rate, ρ.

A key property of this model is obtained by Pissarides by differentiating (42) with respect to θ and ρ:

$$\frac{\mathrm{d}\theta}{\mathrm{d}\rho} = \frac{c/q(\theta)}{\beta H\zeta/(1-\zeta) - (s + r - \rho)q'(\theta)/[q(\theta)]^2} > 0 \tag{43}$$

thanks to (19) and the assumption that $\rho < r$. Hence by comparing (39) with (42) the following result holds:

PROPOSITION 3 *If the productivity parameter, p, and thus firms' hiring costs, pc increases with time then the equilibrium market tightness is higher and the equilibrium unemployment rate is lower when hiring cost increase: $\theta^{\diamond} > \theta^*$ and $u^{\diamond} < u^*$.*

Moreover from equations (43) and (26) we obtain the following result:

PROPOSITION 4 (CAPITALIZATION EFFECT) *If ρ increases then equilibrium labour-market tightness $\theta^\diamond$ increases and the equilibrium unemployment rate $u^\diamond$ decreases.*

This result has explained by Pissarides as follows: hiring costs, pc, increase with time at rate ρ (from (40)).[10] Thus there is a convenience for a firm in anticipating hirings. In other terms for every 100 euros of future hiring costs anticipated by a firm

- it 'loses' r euros as opportunity cost;

- it 'gains' ρ euros as saving on future hiring costs.

Hence $r - \rho$ is the 'effective' discount rate for a firm. A faster rate of technological progress decreases the rate at which future income flows are discounted. This increases the present discounted value of future profits arising from creating today new job slots, leading firms to open more vacancies, and thus reducing unemployment. For this reason this effect has been called 'capitalization effect' by Aghion and Howitt (1994, p. 478).[11]

These conclusions on the effects of growth on unemployment obtained in propositions 3 and 4 seem however be based on a rather unlikely consequence of increase in productivity. This is an example of how the most abstruse arguments and conclusions are sometimes passed off, just enough if presented through the mythical figure of 'microfoundation'.

3.2.6 Ramsey preferences

It is puzzling to see how these conclusions may be easily be upset if the consumers' behaviour is endogenized. This exercise has been put forward by Eriksson (1997): he supposes that households have 'Ramsey preferences', that is, that intertemporal preferences of the generic household j are described by the function $\int_0^{+\infty}[c_j^{1-\gamma}/(1-\gamma)]e^{-r_j t}\mathrm{d}t$, where $1/\gamma$ is the elasticity of substitution between consumption at any two points in time and r_j is the individual discount rate. With this amendment he obtains that $\mathrm{sign}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\rho}\right) = -\mathrm{sign}\left(\frac{\gamma}{1-\chi} - 1\right)$, where χ is the tax rate on capital income. Thus by assuming $\gamma > 1 - \chi$, that is, that the elasticity of substitution is sufficiently small,[12] he obtains that

$$\frac{\mathrm{d}\theta}{\mathrm{d}\rho} < 0 \quad \text{and} \quad \frac{\mathrm{d}u}{\mathrm{d}\rho} > 0$$

(obviously the inequalities are reversed if $\gamma < 1 - \chi$ is supposed). We have thus a positive relationship between growth and unemployment, the opposite of the Pissarides' capitalization effect:

[10] This assumption seems rather questionable, as Pissarides supposes that "[a]s in standard neoclassical model, technological progress is 'disembodied,' in the sense that all existing and new jobs benefit from the higher labour productivity without the need to replace their capital stock." Pissarides (2000, p. 75)

[11] Similar conclusions are drawn by Pissarides (2000, § 3.4) by introducing some *ad hoc* assumptions with reference to a model where the interest rate is endogenously determined.

[12] Eriksson introduces this assumption as "from an empirical point of view it should not be controversial" (see Eriksson (1997, pp. 81-2)).

PROPOSITION 5 (ERIKSSON) *If ρ increases then equilibrium labour-market tightness θ decreases and the equilibrium unemployment rate u increases.*

This proposition is a challenge for the search model in a long-run perspective: it is sufficient to introduce a C.E.S. utility function and a 'plausible' assumption on its parameter that the capitalization effect breaks down!

3.2.7 Concluding remarks

This for what concerns the capitalization effect. But also the explanation of the permanence of unemployment in the long run equilibrium is less convincing. Actually in this context unemployment is the consequence, on one side, of *random adverse shocks* on the labour market and, on the other side, of the impossibility for the system to adjust immediately to a new equilibrium, owing to frictions, *exogenously* introduced through a *matching function*. Unemployment is the 'equilibrium' reaction of the system to *exogenous* causes. All the effort done by the theory has been the formulation of a 'rational' behaviour of firms and workers against frictions and shocks. But these frictions and shocks can only be considered *short-run* causes of unemployment. For they can explain *long-run* unemployment it has been necessary to suppose an unceasing occurrence of these shocks, but *no attention* has been paid to explain *why* these shocks occur. We have thus a rigorous microfoundation, but a short economic explanation. This is the opposite of what happens in some non-orthodox theories, like those presented later. But before to move in this directions it is worth to consider another significant research program on growth, that moves within the orthodox approach, but that goes deeply at the roots of the increases of productivity and obtain more robust conclusions: it's the so called 'neo-Shumpeterian' approach put foreword by Aghion and Howitt.

3.3 The 'creative-distruction' approach

3.3.1 'Creative-distruction' effect *versus* 'capitalization' effect

In a paper published four years later the first edition of the Pissarides (1990) book, Aghion and Howitt (1994) point out that "[b]oth Phelps and Pissarides [...] model the growth process in a way that ignores its reallocative aspect. More specifically they assume that productivity rises equally rapidly in all jobs, existing and potential, as might be the case if it resulted from broadly based increases in human capital, rather than assuming that productivity increases are embodied in new jobs at the expense of old jobs." Aghion and Howitt (1994, p. 478)

On this basis they focus upon the idea that growth arise "explicitly form the introduction of new technologies that require labour re-allocation for their implementation, thereby endogenizing the job-destruction rate which the Pissarides model takes as given." Aghion and Howitt (1994, p. 478) They present their ideas through a significant variant of the search model presented by Pissarides and proves that the *creative destruction* effect dominates the *capitalization* effect.

3.3.2 The model

There is a continuum of firm in the economy; the total mass is endogenously determined in steady state-equilibrium. Each firm, which is infinitely-lived, is thought as 'research facility' for producing new knowledge. Let D_t the sunk cost of setting-up a research facility at date t; once D_t has been sunk, each firm generates a stream of innovations, according to a Poisson process, with parameter ν. New ideas are embodied into new plants in order to be implemented. Production of the final good of the economy takes place at any point in time within a continuum of 'production units'; each production unit consists of three elements: (a) a plant embodying a technology of some vintage t; (b) a worker, appropriately matched with the machine; and (c) a (variable) amount of human capital, x. The output flow at any time s of a production unit based on a plant introduced at date t is

$$y_s = A_t \psi(x_s - a), \tag{44}$$

where ψ is a well-behaved production function, with $\psi(0) = 0$, $\psi' > 0$, $\psi'' < 0$, $\psi'(0) = +\infty$ and $\lim_{z \to +\infty} \psi'(z) = 0$; $a \geq 0$ is the minimum human capital input and $A_t = A_0 \cdot e^{\rho t}$ is the unit's productivity parameter. In this presentation we consider parameter ρ, i.e. the growth rate of the parameter of the leading technology, A_t, as exogenous.[13]

When an innovation occurs in a given firm at date t it open access of that firm to the leading technology A_t as of that date. Provided that the firm builds the machine that embodies the innovation at the implementation cost C_t it will be able to produce output according to function (44).

Consider now the labour side. The aggregate flow of new matches, mL, is a function of the mass of searching workers, i.e. the labour force, L, and of the mass of vacancies, vL, hence $mL = m(L, vL)$. To simplify the analysis the matching process between firms and workers is supposed deterministic. A firm that has just experimented an innovation spends $1/q$ units of time before matching with a worker whose skills are adapted to the machine embodying the innovation; a worker has to wait $1/p$ units of time to match with the machine. Workers able to work on obsolete machines cannot be matched to new vintages within the same firm. q and p are determined by the matching process. We consider only steady states, in which L is constant; normalize it at 1. Hence

$$q = q(v) = \frac{m(1, v)}{v} \quad \text{and} \quad p = p(v) = \frac{m(1, v)}{1}.$$

Assume that q and p are two differentiable functions of v; assume, moreover, that

$$\begin{aligned} & q(v) > 0 \quad \text{and} \quad p(v) > 0 \quad \text{for all } v, \\ & \lim_{v \to +\infty} q(v) = p(0) = 0 \quad \text{and} \quad q(0) = \lim_{v \to +\infty} p(v) = +\infty, \\ & q'(v) < 0, \quad p'(v) > 0 \quad \text{and} \quad 0 \leq p'(v) \leq p(v)/v \quad \text{for all } v. \end{aligned} \tag{45}$$

[13] An 'endogenous' growth version of the model is also provided by the authors; see Aghion and Howitt (1994, § 3.2)

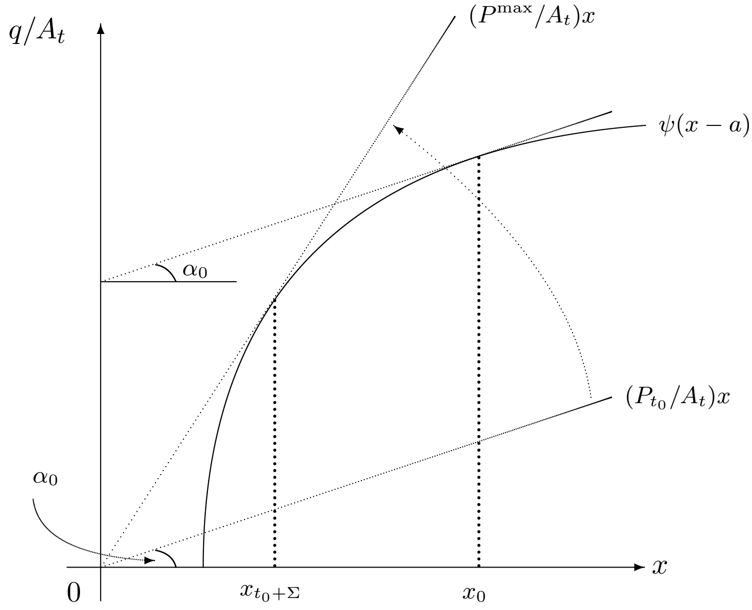

Figure 3: Equilibrium of a production unit

Each worker continues searching for a better job. Let Σ the duration of the match. Assume that

$$1/p(v) > \Sigma;$$

this condition guarantees a positive amount of involuntary unemployment in steady state. More precisely: the flow of worker *into* unemployment is given by the frequency of production units' obsolescence, $1/\Sigma$, times the number of units currently producing, $1-u$;[14] the flow of worker *out of* employment is the job finding rate, $p(v)$. In steady state $(1-u)/\Sigma = p(v)$, that is,

$$(BC) \qquad u = 1 - \Sigma p(v). \tag{46}$$

This is, as in Pissarides, the Beveridge curve (BC): it is a non-increasing function of v as $\mathrm{d}u/\mathrm{d}v = -\Sigma p'(v) \geq 0$.

Σ can be determined endogenously by the model after having analyzed the behaviour of each unit. When an innovation occurs in a given firm at time t and this firm decides to implement this innovation, he has to wait date $t_0 = t + 1/q$ to match a suitably skilled worker. Then the flow generated by this production unit at any date $\tau \geq t_0$ is:

$$\max_{x \geq a}[A_t\psi(x-a) - P_\tau x] =: \Pi(P_\tau) = A_t \cdot \Pi(P_\tau/A_t), \tag{47}$$

where $\Pi'(\cdot) < 0$. The solution for each date τ of (47) is given geometrically by that level of human capital where $\psi(x-a)$ curve has slope equal to that of (P_τ/A_t): for example,

[14]Remember that each unit employs one worker and that labour force is normalized at 1.

at t_0 $P_\tau = P_{t_0}$ and thus the optimal correspondent amount of human capital is $x_\tau = x_0$, like in figure 3. But A_t remains constant for that unit, while P_τ grows at rate ρ, which is the rate of growth of A_t for the rest of the economy, $P_\tau = P_0 e^{\rho\tau}$. This makes sense in this context as innovations are embodied into new machines and workers must be more and more skilled to use them. Then as P_τ growth, x_τ decreases until to $x_{t_0+\Sigma}$, where the unit becomes unprofitable; this happen at date $t_0 + \Sigma$, when P_τ/A_t reaches the shut-down value, $P^{\max}/A_t = \Pi^{-1}(0)$. $P^{\max}$ is individuated by the couple of conditions:

$$\psi(x - a) = (P^{\max}/A_t)x \tag{48a}$$

$$\psi'(x - a) = (P^{\max}/A_t) \tag{48b}$$

($\psi(x - a)$ must intersect $(P^{\max}/A_t)x$—eq. (48a)—with the same slope—eq. (48b)). (48) is a system of two equations in two unknowns, $P^{\max}$ and $x_{t_0+\Sigma}$. Once $P^{\max}$ is known, we can calculate Σ by the equality:

$$P_{t_0+\Sigma} \equiv P_{t_0} e^{\rho\Sigma} = P^{\max}, \tag{49}$$

or, equivalently,

$$\Sigma = \Gamma/\rho, \quad \text{where} \quad \Gamma := \ln P^{\max} - \ln P_{t_0} > 0. \tag{50}$$

Obviously, the higher the growth rate ρ of the price of human capital, the sooner the production unit will become unprofitable (i.e. the lower the duration of a match Σ will be). Aghion and Howitt emphasize this inverse relationship between ρ and Σ and call it *direct creative destruction effect* of growth on employment. Aghion and Howitt (1994, p. 482) This effect may be seen directly from (BC) after inserting (50) into (46).

$$u = 1 - \Gamma p(v)/\rho. \tag{46'}$$

Thus, for given Γ and v, an increase in ρ directly raises the job-destruction rate, $1/\Sigma$, increasing thus the unemployment rate.

In order to study the indirect effects of growth on unemployment we need to study the conditions that determine v and Γ: we will obtain two additional equations, the first one which consists in a free-entry condition for firms at any date t and the second one which is a market clearing condition for human capital.

For the first condition consider the entry decision for a firm at a given date t. Suppose that the sunk *cost* of entry ad date t is proportional to A_t, i.e. $D_t = d \cdot A_t$. For such a firm the innovation is supposed to arrive at date $t + \vartheta$, where ϑ is a Poisson random variable with rate ν. The expected net *benefit* of entry at date t, W_t, is thus given by: i) the present value of the expected profit stream accruing from the innovation arriving at $t + \vartheta$, $V_{t+\vartheta}$; ii) the present value of the expected net benefit that will arise from $t + \vartheta$ on, $W_{t+\vartheta}$:

$$W_t = E_{\vartheta \geq 0}[(V_{t+\vartheta} + W_{t+\vartheta})e^{-r\vartheta}]. \tag{51}$$

Suppose that both W and V are proportional to A_t, hence:

$$\begin{aligned} W_t &= W \cdot A_t = W_0 \cdot e^{\rho t}, \quad \text{where} \quad W_0 := W \cdot A_0 \quad \text{and} \\ V_t &= V \cdot A_t = V_0 \cdot e^{\rho t}, \quad \text{where} \quad V_0 := V \cdot A_0 \end{aligned} \tag{52}$$

By substituting (52) into (51) we obtain:

$$\begin{aligned} W \cdot A_t &= E_{\vartheta \geq 0}[(V + W) \cdot A_{t+\vartheta} \cdot e^{-r\vartheta}] = \\ &= A_t \cdot E_{\vartheta \geq 0}[(V + W) \cdot e^{-(r-\rho)\vartheta}], \end{aligned}$$

that is,

$$\begin{aligned} W &= E_{\vartheta \geq 0}[(V + W) \cdot e^{-(r-\rho)\vartheta}] = \\ &= (V + W) \int_0^{+\infty} e^{-(r-\rho)\vartheta} \cdot \nu e^{-\nu\vartheta} \mathrm{d}\vartheta = \\ &= (V + W) \cdot \frac{\nu}{\nu + r - \rho}; \end{aligned}$$

hence

$$W = \frac{\nu V}{r - \rho}.$$

Thus by comparing sunk costs, D_t and benefits, W_t, we obtain the free-entry condition, $D_t = W_t$, that is, $d = W$, that is,

$$d = \nu V/(r - \rho). \tag{53}$$

We have now to obtain an expression for V: an innovation occurring at date t begins to produce its output only after the matching with an appropriate worker, that is, as from $t + 1/q(v)$. At that date the cost C_t will be paid to implement the production unit embodying this innovation; it is supposed that $C_t = c \cdot A_t$. From date $t + 1/q(v)$ to date $t + 1/q(v) + \Sigma$ the production unit will generate a flow of surplus at each date σ given by $\Pi(P_{t_0+\sigma})$; thanks to (49) we can write

$$P_{t_0+\sigma} = P^{\max} \cdot e^{\rho\sigma - \Gamma}, \quad \text{for } \sigma \leq \Gamma/\rho = \Sigma. \tag{54}$$

Hence the present value of flows $\Pi(P_{t_0+\sigma})$ may be written as

$$e^{-r/q(v)} \cdot \int_0^{\Gamma/\rho} e^{-r\sigma} \Pi(P^{\max} \cdot e^{\rho\sigma - \Gamma}) \mathrm{d}\sigma,$$

where the first exponential is explained by observing that this surplus will flow $1/q(v)$ instant after t. The *net* surplus of the production unit is obtained after deducing the implementation cost. As in Pissarides the firm bargain to obtain a constant fraction β of this surplus; hence

$$V_t = V \cdot A_t = e^{-r/q(v)} \left\{ \beta \int_0^{\Gamma/\rho} e^{-r\sigma} \Pi(P^{\max} \cdot e^{\rho\sigma - \Gamma}) \mathrm{d}\sigma - c \cdot A_t \right\}$$

hence

$$V = e^{-r/q(v)} \left\{ \beta \int_0^{\Gamma/\rho} e^{-r\sigma} \Pi[(P^{\max}/A_t) \cdot e^{\rho\sigma-\Gamma}] \mathrm{d}\sigma - c \right\},$$

which, substituted into (53) gives the expression of the free-entry condition for firm:

$$(FE) \qquad d = \frac{\nu}{r-\rho} \cdot e^{-r/q(v)} \left\{ \beta \int_0^{\Gamma/\rho} e^{-r\sigma} \Pi[(P^{\max}/A_\sigma) \cdot e^{\rho\sigma-\Gamma}] \mathrm{d}\sigma - c \right\}. \qquad (53')$$

In order to study the properties of the link between v and ρ according to equation(53′) observe, first of all, that $(P^{\max}/A_\sigma) \cdot e^{\rho\sigma-\Gamma} = (P^{\max}/A_0) \cdot e^{-\rho\sigma+\rho\sigma-\Gamma} = (P^{\max}/A_0) \cdot e^{\Gamma}$ does not depend on ρ; thus the argument of the integral is constant with respect to ρ; hence ρ may affect v (and thus u) through: i) the fraction $\nu/(r-\rho)$ and ii) through the superior extreme of the integral, Γ/ρ. Consider an *increase in the rate of growth.* For what concerns effect i) it reduces the net discount rate, $r-\rho$, at which firms capitalize their expected income; this increases the present benefit of entry and this increases the equilibrium level of vacancies, v;[15] this, through equation (46′), decreases *the equilibrium unemployment rate*, u. This is the same *capitalization effect* obtained by Pissarides. For what concerns effect ii) the increase in ρ reduces the lifetime of the production unit, $\Sigma = \Gamma/\rho$, since the price of the human capital that has to be used to take advantage of that innovations grows; this reduces the value of V of each innovation and thus reduces firms' incentives to enter and open new vacancies.[16] Thus through channel ii) the equilibrium unemployment rate increases. This *indirect creative destruction effect* works in the same direction of the direct creative destruction effect previously emphasized (see at p. 22).

We have to complete the model with the market clearing condition on the market for human capital. Let $x(P_{t_0+\sigma}) = x(P^{\max} \cdot e^{\rho\sigma-\Gamma})$ the solution at σ to problem (47) i.e. the optimal demand of human capital at time σ; the total demand for capital by any production unit during its lifetime is $\int_0^{\Gamma/\rho} x(P^{\max} \cdot e^{\rho\sigma-\Gamma})\mathrm{d}\sigma$. The average demand for human capital during the lifetime of each productive unit is

$$\frac{1}{\Gamma/\rho} \int_0^{\Gamma/\rho} x(P^{\max} \cdot e^{\rho\sigma-\Gamma})\mathrm{d}\sigma = \frac{1}{\Gamma} \int_0^{\Gamma/\rho} x(P^{\max} \cdot e^{\varsigma-\Gamma})\mathrm{d}\varsigma \quad \text{where } \varsigma := \rho\sigma.$$

Remembering that $(1-u)$ is the number of operating productive units the market clearing condition on this market is thus

$$(H) \qquad X = (1-u)\frac{1}{\Gamma} \int_0^{\Gamma/\rho} x(P^{\max} \cdot e^{\varsigma-\Gamma})\mathrm{d}\varsigma, \qquad (55)$$

where $X > 0$ is the aggregate supply of human capital. By the law of large numbers the r.h.s. of (55) is also the aggregate demand for human capital at any point of time.

[15]In analytical terms an increase in ρ increases $\nu/(r-\rho)$; thus for the r.h.s. of (53′) remain constant $e^{-r/q(v)}$ must decrease, and this happens if v increases.

[16]In order to keep the r.h.s of (53′) constant after the reduction of the integral it is necessary that exponential $e^{-r/q(v)}$ increases; this happens if v decreases.

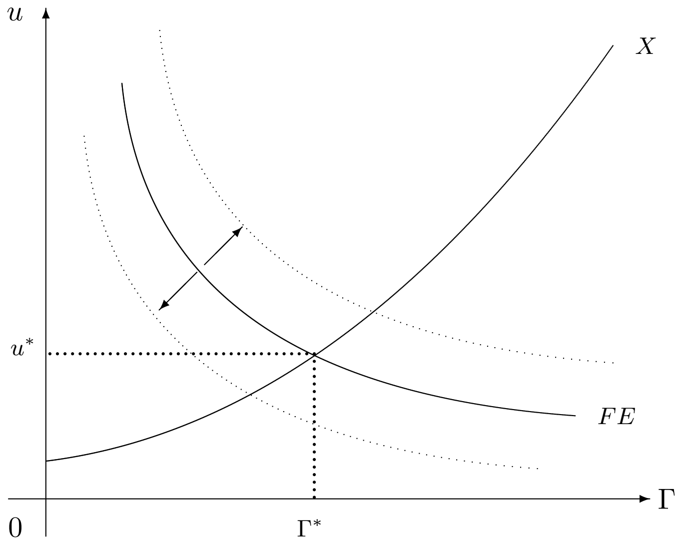

Figure 4: Steady state equilibrium

We have thus three equations, (46), ($53'$) and (55) in three unknowns, u, v and Γ.

The authors prove that under some regularity assumption of functions $\Pi(\cdot)$ and $x(\cdot)$, in addition to the other assumption of the model, there exists a unique steady-state equilibrium, (u^*, v^*, Γ^*) Aghion and Howitt (1994, p. 484).

We have seen before that when ρ change three effects take place: i) the direct creative destruction effect; ii) the indirect creative destruction effect; iii) the capitalization effect. We have now all the elements to study the *total* effect that will actually take place. To this purpose we can observe that equation ($53'$) draws an increasing relationship between v and Γ: in fact as the argument of the integral is non-negative an increase in Γ extends the area below the curve; thus for the r.h.s. of ($53'$) remain constant $e^{-r/q(v)}$ must decrease, and this happens if v increases; let $v = FE(\Gamma)$ this curve, with $FE'(\Gamma) > 0$. Substituting this expression into (46) we obtain

$$u = 1 - \frac{\Gamma p[FE(\Gamma)]}{\rho} = BC(\Gamma), \quad \text{with} \quad BC'(\Gamma) < 0, \tag{46$'$}$$

where the sign of the derivative descends from the fact that $p(v)$ is an increasing function (see (45)). Thus ($46'$) summarizes equations BC and FE. On the other side equation (55) entails an increasing relation between u and Γ: in fact it can be proved that $\frac{1}{\Gamma}\int_0^{\Gamma/\rho} x(P^{\max} \cdot e^{\varsigma-\Gamma})d\varsigma$ is an increasing function of Γ;[17] hence when Γ increases to keep the r.h.s. constant at X, u must increase:

$$u = X(\Gamma), \quad \text{with} \quad X'(\Gamma) > 0. \tag{55$'$}$$

We can represent on the same diagrams curves BC and H (see figure 4).

[17]See, Aghion and Howitt (1994, p. 484, fn. 16).

When ρ changes curve X remains unchanged. It is not possible to say *a priori* in which direction curve BC move. Aghion and Howitt prove the following:[18]

PROPOSITION 6 (AHGHION-HOWITT) *For any given $\rho_0 \in (0, r)$ let $v_0 = v^*(\rho_0)$ the corresponding solution of system (46), (53′) and (55). We have that:*

1. $\left.\frac{\mathrm{d}u^*}{\mathrm{d}\rho}\right|_{\rho=\rho_0} > 0$, *provided that either d or $p'(v_0)$ are sufficiently small;*

2. *for a given entry cost d we have* $\left.\frac{\mathrm{d}u^*}{\mathrm{d}\rho}\right|_{\rho=\rho_0} < 0$, *for g sufficiently close to r and c sufficiently small.*

Observe that when item 1 holds the creative destruction effect prevails. This happens in two cases. i) When d is sufficiently small; to understand consider the extreme case in which $d = 0$; in this case the term that gives rise to the capitalization effect disappears, in fact FE becomes

$$\beta \int_0^{\Gamma/\rho} e^{-r\sigma}\Pi[(P^{\max}/A_\sigma) \cdot e^{\rho\sigma-\Gamma}]\mathrm{d}\sigma = c.$$

The creative destruction effect prevails also if $p'(v_0)$ is very small; again, to understand, consider the extreme case in which $p'(v_0)$ remains constant: (46′) becomes $u = 1 - \Gamma\bar{p}/\rho$ and if ρ increases, it rotates unambiguously outward, increasing thus the equilibrium unemployment rate.

When item 2 holds the capitalization effect prevails. This happens when c is small, so that the discounted net value of an innovation becomes sufficiently large, and when $\rho \to r$. Aghion and Howitt suggest that the the link between the equilibrium unemployment rate, u^*, and the the growth rate, ρ may be thought as an inverted U-shaped function (see figure 5).

The analysis of Aghion and Howitt permitted us to grasp fully the main phenomena that underneath the relation between growth and unemployment. It has caught aspects that were excluded in the Pissarides analysis.[19] Anyway the search model and the neo-Schumpeterian model constitute the major frameworks through which neoclassical theory can manage the study of unemployment within long-run growth analysis.

Alongside these mainstream analysis we can also find several heterodox approaches to this topic; they are founded of different methodological assumptions, on different visions of the working of an economic system and on different analytical framework to describe the economic system. Some orthodox scholars may found them less rigorous from the

[18] See Aghion and Howitt (1994, Appendix).

[19] This is true for Pissarides (1990). In the second edition of the book, Pissarides (2000) included the treatment of the phenomenon of endogenous job destruction analyzed by Aghion and Howitt (1994).

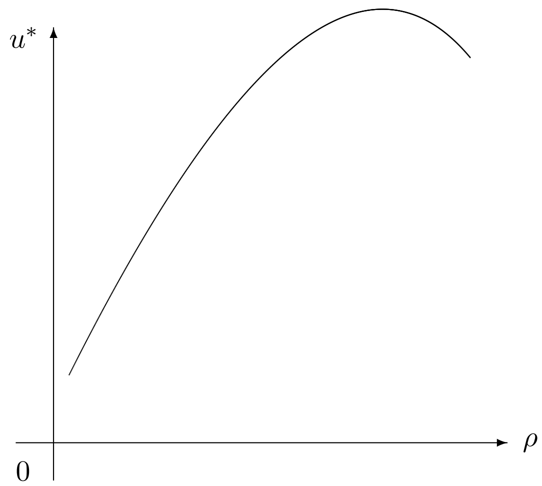

Figure 5: Equilibrium unemployment rate when ρ varies

theoretical point of view, because of the fact that the relationships on which they are built up are not obtained by an optimal choice problems, in other words they are not 'micro-founded'. Such approach is seen by someone a serious limitation: it is said that the main assumptions are introduced *ad hoc* in relation to the results that have to be obtained. In some cases it can be said that these relationships are introduced on the basis of the common sense. This is a methodological aspect that will not be discussed here. In the next section we will present two analytical non-orthodox frameworks that will provide us two different views of the links between growth and unemployment. Their relative analytical simplicity, together with their explanatory power of several real phenomena, justify the relevance which the literature recognize them.

The first approach focuses upon the interactions between growth and cycle: it has been originated by the seminal works of Goodwin (1967) and of Akerlof and Stiglitz (1969). The second approach will focus upon the consequence of structural change of an economic system and has been originated by Pasinetti (1965), (1981) and (1993).

4 Non-orthodox approaches

4.1 The growth-cycle approach

This line of analysis has been put forward, independently, by Goodwin (1967) and by Akerlof and Stiglitz (1969);[20] later van der Ploeg carries out a generalization of this contributions: in (1983b) reproposes a framework analogous to Akerlof and Stiglitz; in (1983a) provide a growth-cycle model along post-Keynesian lines.

[20]Both these works were presented in 1965, the Goodwin's paper at the Rome First World Congress of Econometric Society, as referred by Goodwin himself in a footnote at p. 54 of his paper, the Akerlof-Stiglitz's paper at the New York Meeting of the Econometric Society, as referred by Balducci and Candela (1982, p. 17, f. 5).

4.1.1 The Goodwin-Akerlof-Stiglitz approach

We recall briefly the basic structure of this framework by following the Akerlof-Stiglitz paper, which received apparently less evidence than the Goodwin's one.

CONSTANT AMPLITUDE OSCILLATION

There are three building blocks of this models:

- *Wage-employment relationship.* The rate of change of real wages is a decreasing function of the level of unemployment, according to a sort of *real*-Phillips curve:

$$\frac{\dot{w}}{w} = Z(u), \quad Z' < 0, \quad Z(1) < 0, \quad Z(0) > 0. \tag{56}$$

- *Savings-Investments relationship.* It is assumed that savings, S, equal investments ex-ante, that is,

$$S = \dot{K}. \tag{57}$$

  Savings are a constant fraction s_w of wages, W, and a constant fraction s_p of profits, Π:

$$S = s_w W + s_p \Pi, \quad \text{with} \quad s_w \leq s_p. \tag{58}$$

- *Technology.* It is normally represented by a well-behaved aggregated production function,

$$Q = F(K, E), \tag{59}$$

  A first case is obtained by supposing a fixed output/capital ratio, $Q/K = q$, and a fixed employment/capital ratio,

$$E/K = b \ (= 1/k). \tag{60}$$

  As we are considering an economy in which there can be unemployment, it is useful to recall the attention on the distinction between the capital/employment ratio, k, defined in (60), and the capital/labour ratio,

$$K/N = \kappa, \tag{61}$$

  where N is the labour force. If $N = E$ there is full-employment and $\kappa = k$; otherwise $E < N$ and $\kappa < k$.

By combining equations (56)-(61) we obtain the following differential equation system:

$$\dot{\kappa} = s_p q \kappa - (s_p - s_w) w b \kappa - n\kappa \tag{62a}$$

$$\dot{w} = w Z(1 - b\kappa), \tag{62b}$$

where $n = \dot{N}/N$ is given exogenously and we suppose, by simplicity, $\rho = 0.$. The unemployment rate is related to the state variable κ by:

$$u = \frac{N-E}{N} = 1 - \frac{E}{K} \cdot \frac{K}{N} = 1 - b\kappa.$$

The equilibrium values of the state variable of system (62) are given by:

$$w = w^* = \frac{s_p q - n}{(s_p - s_w)b} \quad \text{and} \quad \kappa = \kappa^*, \quad \text{where } \kappa^* \text{ is such that } Z(1 - b\kappa^*) = 0;$$

the correspondent equilibrium value of the unemployment rate is:

$$u = u^* = 1 - b\kappa^*.$$

Thanks to the assumptions on $Z(\cdot)$ we have that $0 < u^* < 1$.

The Jacobian matrix of system (62) evaluated at (κ^*, w^*),

$$\mathbf{J}^* = \begin{bmatrix} 0 & -(s_p - s_w)b\kappa^* \\ -w^* Z'^* b & 0 \end{bmatrix},$$

has two purely imaginary eigenvalues, $\eta = \pm i\sqrt{-b^2 Z'^*(s_p - s_w)w^*\kappa^*}$. This means that w, κ and u have constant amplitude oscillations around the equilibrium. These results may be considered common both in Goodwin and in Akerlof-Stiglitz analysis.

Dampened oscillation

Akerlof and Stiglitz went further, considering also the case of smooth substitutability among capital and labour; capitalists chose the optimal input combination by maximizing profits; in term of employed workers the corresponding first order condition is

$$w = f(k) - kf'(k);$$

by inverting we obtain:

$$k = k(w), \qquad \text{with } k'(w) = -1/[kf''(k)] > 0 \tag{63}$$

In this economy the capital/labour ratio κ varies as follows:

$$\dot{\kappa} = \frac{\dot{K}N - \dot{N}K}{N^2} = s_p f(k)\frac{r}{k} - (s_p - s_w)w\frac{\kappa}{k} - n\kappa.$$

Considering (63) we obtain:

$$\dot{\kappa} = \kappa \left\{ \frac{s_w f[k(w)]}{k(w)} + (s_p - s_w)f'[k(w)] - n \right\}, \tag{64a}$$

that together with the law of evolution of the wage rate

$$\dot{w} = w \cdot Z\left(1 - \frac{\kappa}{k(w)}\right) \tag{64b}$$

constitutes a second order differential equation system. The equilibrium of this system is given by:

$$w = w^{\diamondsuit}, \quad \text{and} \quad \kappa = \kappa^{\diamondsuit},$$

where $w^{\diamondsuit}$ and $\kappa^{\diamondsuit}$ are such that $s_w f[k(w^{\diamondsuit})]/k(w^{\diamondsuit}) + (s_p - s_w)f'[k(w^{\diamondsuit})] - n = n = 0$ and $Z(1 - \kappa^{\diamondsuit}/h(w^{\diamondsuit})) = 0$.

The characteristic equation of the Jacobian matrix of this system, evaluated at $(\kappa^{\diamondsuit}, w^{\diamondsuit})$ is:

$$\eta^2 - \frac{Z'^{\diamondsuit}\kappa^{\diamondsuit}k'(w^{\diamondsuit})}{(k^{\diamondsuit})^2}w^{\diamondsuit}\eta - w^{\diamondsuit}\kappa^{\diamondsuit}k'(w^{\diamondsuit})\frac{Z'^{\diamondsuit}}{k^{\diamondsuit}} \cdot \left[\frac{s_w w^{\diamondsuit}}{(k^{\diamondsuit})^2} - (s_p - s_w)f''(k^{\diamondsuit})\right] = 0,$$

where $k^{\diamondsuit} = k(w^{\diamondsuit})$. As all coefficients of this equations are positive the equilibrium is locally asymptotically stable (cfr. Gandolfo (1997, p. 198)). Eigenvalues are complex if the following condition holds:[21]

$$Z'^{\diamondsuit} > 4f''(k^{\diamondsuit}) \cdot \frac{(s_p - s_w)(k^{\diamondsuit})^3 - s_w w^{\diamondsuit}/f''(k^{\diamondsuit})}{\kappa^{\diamondsuit}w^{\diamondsuit}k'(w^{\diamondsuit})} \tag{65}$$

Thus if we allow for substitution between factors, w, κ (and u) converge (monotonically or with oscillations) towards their equilibrium values.

4.1.2 Post-Keynesian dynamics (van der Ploeg)

THE MODEL

A similar set of results has been obtained by van der Ploeg (1983b) with an alternative model elaborated along post-Keynesian lines. The main assumptions characterizing this framework are the following:

- The production possibilities are described by shaping a flexible capital/output ratio as follows:

$$\frac{K^*}{Q} = v = v^* \cdot \left(\frac{E}{L}\right)^{-\eta_1} \cdot \left[\left(\frac{\Pi}{PK^*}\right)_e\right]^{\eta_2}, \quad \eta_1 > 0, \quad \eta_2 > 0: \tag{66}$$

  capital grows with respect to output in periods of low employment and with high expected rate of return.

- The supply of labour per head is supposed to be a decreasing function of the real wealth per head, that is, the ratio K^*/N (real wealth is constituted by the amount of capital of the system):

$$l := \frac{L}{N} = \mu_0 \left(\frac{K^*}{N}\right)^{-\mu}, \quad \mu \geq 0. \tag{67}$$

[21] Condition (65) is different from the condition obtained by authors, in footnote 2 at p. 273.

- Technical progress is described by a Kaldor-type function:

$$\frac{\dot{Q}}{Q} - \frac{\dot{E}}{E} = \phi\left(\frac{\dot{K}^*}{K^*} - \frac{\dot{E}}{E}\right), \quad \phi(0) > 0, \quad \phi' > 0, \quad \phi'' < 0. \tag{68}$$

- Capialists and workers have different saving propensities:

$$sY = s_p\Pi + s_w WE, \quad \text{with } s_p > s_w. \tag{69}$$

- Real savings are immediately invested:

$$\dot{K}^* = S/P = I - \delta K^*. \tag{70}$$

- Consumption depends positively on income and on wealth and savings are determine investments:

$$PC = (1 - s)Y + \gamma PK^*, \quad \gamma > 0, \tag{71}$$

- The evolution of population is driven by a Gompertz process:

$$n(t) = \frac{\dot{N}}{N} = \nu_1 + \nu_2 e^{-\nu_2 t}, \quad \nu_2 \geq 0. \tag{72}$$

- The dynamics of the wage rate is described by an augmented Phillips curve

$$\frac{\dot{W}}{W} = \beta_1\left[\left(\frac{u}{u_0}\right)^{-\rho} - 1\right] + \beta_2\left(\frac{\dot{E}}{E} - \frac{\dot{L}}{L}\right) + \beta_3 p^e + \beta_4 + \beta_5\left(\frac{\dot{Q}}{Q} - \frac{\dot{E}}{E}\right), \tag{73}$$

  Where p_e is expected rate of price inflation. The various element of this formula will be explained in the course of the model.

It is worth to be noted that in this model the dynamics of prices is considered explicitly: contrarily to what has been made in the two models seen before in this framework the Phillips curve appears in nominal, rather in real, form. Thus we have to explain the actual and the expected price level. For the *actual* price levels van der Ploeg recalls two complementary theories:

Cost-push inflation: producers fix prices to yield a desired share of profit, Δ: thus price are given by $P^* = (1 + \mu)WE/Q$, where the mark-up rate, μ, is fixed in order to guarantee that $(P^*Q - WE)/(P^*Q) = \Delta$. In this way $P^* = \frac{1}{1-\Delta}\frac{WE}{Q}$ and the rate of cost-push inflation is given by:

$$p_c = \frac{1}{\tau_p}\left(\frac{P^* - P}{P}\right) = \frac{1}{\tau_p}\left(\frac{WE}{Y} \cdot \frac{1}{1 - \Delta} - 1\right), \quad \tau_p > 0, \tag{74}$$

where τ_p is the average time between marking up.

Quantity theory: the rate of inflation it is given by:

$$p_m = \frac{\dot{M}^s}{M^s} - \frac{\dot{Q}}{Q} = m^s(t) - \frac{\dot{Q}}{Q}, \tag{75}$$

where $m(t)$ is the exogenous growth in the money supply.

The *expected* rate of price inflation is determined by an adaptive mechanism, so that

$$\frac{\mathrm{d}p^e}{\mathrm{d}t} = \tau_e^{-1}(p - p^e), \quad \tau_e > 0, \tag{76}$$

where τ_e is the adjustment coefficient.

Equations (66)-(76) describe the post-Keynesian dynamic model presented by van der Ploeg (1983a).

Warranted rates of growth

The assumptions on function $\phi(\cdot)$ ensure that a unique fixed point $\rho* = \phi(\rho*)$ exists. From this it is possible to obtain the equilibrium dynamics of the variables of system (66)-(76):

$$\frac{\dot{Q}}{Q} = \frac{\dot{K}^*}{K^*} = \nu_1 + \frac{\rho}{1+\mu} =: g_n, \tag{77}$$

$$\frac{(\dot{E/N})}{(E/N)} = \frac{\dot{l}}{l} = -\mu\frac{\rho}{1+\mu}, \tag{78}$$

$$\frac{(\dot{W/P})}{(W/P)} = \frac{(\dot{Q/E})}{(Q/E)} = \rho. \tag{79}$$

g in (77) is the *natural* rate of growth of production and capital. From the equilibrium conditions on the goods market ($C + I = Q$) we obtain the *warranted* rate of growth:

$$g_w = \frac{\dot{K}^*}{K^*} = \frac{s}{v} - \gamma - \delta.$$

Long-run dynamic equilibrium requires, as known, that $g_w = g_n$. Following the post-Keynesian tradition this equality can be achieved by fixing a particular configuration of income distribution, individuated by the following wage share:

$$WE/Y = (s_p - \bar{s})/(s_p - s_w), \quad \text{where} \quad \bar{s} = v(\gamma + \delta + g_n).$$

The long-run equilibrium rate of unemployment is given by:

$$u = u_0 \left\{ 1 + \frac{1}{\beta_1}\left[(1-\beta_5)\rho + (1-\beta_3)\bar{p} - \beta_4\right] \right\}^{-1/\rho},$$

where $\bar{p}$ is the long-run equilibrium inflation rate.

Dynamics

At this point van der Ploeg analyzes the different dynamical paths arising under different assumptions on technology or on the mechanisms to fix the wage rate.

• *Perpetual Conflict (PC)* - Suppose $\eta_1 = \eta_2 = 0$, that is, a fixed capital/output ratio; abstract for the moment from inflation (this can be done by setting $p^e = p$ and $\beta_3 = 1$ or by considering p and p^e exogenous); suppose, moreover, that $\beta_2 = 0$. The model described by equations (66)-(76) collapses to a Goodwin-type model

$$\frac{\dot{\theta}}{\theta} = \beta_1 \left[\left(\tfrac{1-\epsilon}{u_0} \right)^{-\rho} - 1 \right] + \beta_4 - (1-\beta_5)\rho \tag{80a}$$

$$\frac{\dot{\epsilon}}{\epsilon} = (1+\mu) \left[\tfrac{s_p - (s_p - s_w)\theta}{v} - \gamma - \delta - n(t) \right] - \rho. \tag{80b}$$

The steady state values of θ and ϵ within the PC model are:

$$\theta^*_{\mathrm{PC}} = \tfrac{s_p}{s_p - s_w} - \tfrac{v}{s_p - s_w} \left[\gamma + \delta + n(t) + \tfrac{\rho}{1+\mu} \right] \quad \text{and}$$

$$\epsilon^*_{\mathrm{PC}} = 1 - u_0 \left[1 + \tfrac{(1-\beta_5)\rho + (1-\beta_3)\bar{p} - \beta_4(\infty)}{\beta_2} \right]^{-1/\rho}.$$

The Jacobian matrix of system (80) and the corresponding eigenvalues, both evaluated at $(\theta^*_{\mathrm{PC}}, \epsilon^*_{\mathrm{PC}})$ are, respectively,

$$\mathbf{J}^*_{\mathrm{PC}} = \begin{bmatrix} 0 & B \\ -C & 0 \end{bmatrix}, \quad \text{and} \quad \lambda_{1,2} = \pm i\sqrt{BC},$$

where

$$B := \theta^* \beta_1 \tfrac{\rho}{u_0} \left[\tfrac{(1-\beta_5)\rho + (1-\beta_3)\bar{p} - \beta_4(\infty)}{\beta_1} \right]^{(1+\rho)/\rho} > 0$$

$$C := \epsilon^*(1+\mu)(s_p - s_w)/v > 0.$$

Hence, as expected, θ and ϵ have constant amplitude oscillations around the equilibrium.

• *Fear of redundancies (FR)* - This phenomenon may occur when ϵ decreases and workers are prepared to sacrifice some growth in real wages for fear of redundancies; this is captured by parameter β_2 in (73). The system becomes in this case:

$$\frac{\dot{\theta}}{\theta} = \beta_1 \left[\left(\tfrac{1-\epsilon}{u_0} \right)^{-\rho} - 1 \right] + \beta_2 \frac{\dot{\epsilon}}{\epsilon} + \beta_3 p^e - p + \beta_4 - (1-\beta_5)\rho \tag{81a}$$

$$\frac{\dot{\epsilon}}{\epsilon} = (1+\mu) \left[\tfrac{s_p - (s_p - s_w)\theta}{v} - \gamma - \delta - n(t) \right] - \rho. \tag{81b}$$

The long-run equilibrium of the FR model (81) coincide with that of the PC model: $\theta^*_{\mathrm{FR}} = \theta^*_{\mathrm{PC}}$ and $\epsilon^*_{\mathrm{FR}} = \epsilon^*_{\mathrm{PC}}$; the Jacobian matrix evaluated at $(\theta^*_{\mathrm{FR}}, \epsilon^*_{\mathrm{FR}})$ and the corresponding characteristic equation are, respectively:

$$\mathbf{J}^*_{\mathrm{FR}} = \begin{bmatrix} -A & B \\ -C & 0 \end{bmatrix}, \quad \text{and} \quad \lambda^2 + A\lambda + BC = 0,$$

where

$$A := \theta^* \beta_2 (1+\mu)(s_p - s_w)/v > 0;$$
$$B := (\theta^* \beta_1 \rho)[(1-\epsilon^*)/u_0]^{-\rho-1}/u_0;$$
$$C := e^*(1+\mu)(s_p - s_w)/v > 0.$$

As $A > 0$ and $BC > 0$ the eigenvalues are negative or have negative real part; the long run-equilibrium is thus locally asymptotically stable. The author justifies this result by saying: "The loops in wage formation eventually eliminate perpetual conflict, because a high share of labour reduces the demand for employment and workers anticipate that, in this case, excessive wage claims could threaten jobs. Also, a high share of profits increases recruiting activity, the market responds by granting higher wages claims and this reduces the share of profits again." (van der Ploeg (1983a, p. 260)).

• *Inflation (I)* - Consider now the price dynamics: suppose partial compensation for increases in the cost of living, that is, $0 < \beta_3 < 1$, perfect anticipation of the rate of inflation, that is, $p^e = p$, and suppose, for the moment, that inflation comes only from the cost-push mechanism (CP) (eqn. (74)). Suppose again $\beta_2 = 0$. The reference system is thus given by:

$$\frac{\dot\theta}{\theta} = \beta_1 \left[\left(\tfrac{1-\epsilon}{u_0} \right)^{-\rho} - 1 \right] + (\beta_3 - 1)\tau_p^{-1} \left(\tfrac{\theta}{1-\Delta} - 1 \right) + \beta_4 - (1-\beta_5)\rho \tag{82a}$$

$$\frac{\dot\epsilon}{\epsilon} = (1+\mu) \left[\tfrac{s_p - (s_p - s_w)\theta}{v} - \gamma - \delta - n(t) \right] - \rho. \tag{82b}$$

If alternatively we consider money inflation (MI, eqn. (75)), equation (82a) takes the form:

$$\frac{\dot\theta}{\theta} = \beta_1 \left[\left(\tfrac{1-\epsilon}{u_0} \right)^{-\rho} - 1 \right] + (\beta_3 - 1) \left[m^s(t) - \tfrac{s_p - (s_p - s_w)\theta}{v} + \gamma + \delta \right] + \beta_4 - (1-\beta_5)\rho \tag{82a$'$}$$

Both the Jacobian matrix of system (82a)-(82b) and of system (82a′)-(82b) evaluated at its long-run equilibrium and the correspondent characteristic equation are given respectively by:

$$\mathbf{J}_{\mathrm{I}}^* = \begin{bmatrix} -A & B \\ -C & 0 \end{bmatrix}, \quad \text{and} \quad \lambda^2 + (1-\beta_3)A\lambda + BC = 0,$$

where

$$A_{\mathrm{CP}} := \theta^* \tau_p^{-1}/(1-\Delta) > 0 \quad \text{for system (82a)-(82b), or}$$
$$A_{\mathrm{MI}} := \theta^*(s_p - s_w)/v > 0 \quad \text{for system (82a}'\text{)-(82b);}$$
$$B := \theta^* \beta_1 \rho [(1-\epsilon^*)/u_0]^{-\rho-1}/u_0 > 0;$$
$$C := e^*(1+\mu)(s_p - s_w)/v > 0.$$

The eigenvalues are negative if all the coefficients of the characteristic polynomial are positive; this happen if

$$\beta_3 < 1,$$

that is, the economy is locally asymptotically stable under money illusion; moreover we have monotonic convergence towards the equilibrium for $0 < \beta_3 \leq 1 - 2\sqrt{BC}/A$, and dampened oscillation for $1 - 2\sqrt{BC}/A < \beta_3 < 1$.

We can observe thus that the constant amplitude oscillations of the perpetual conflict models are dampened and convergence is assured if a sort of accommodating behaviour from workers is introduced: or fear of redundancies or money illusion. Consider now the case of a flexible K^*/Q ratio.

• *Flexibility in the K^*/Q ratio* - Continue to assume that production requires a fixed capital/output ratio. Oscillations in demand are faced by varying the rate of utilization of capital, $x = K/K^*$. Thus we have:

$$\begin{aligned}
&K/Q = v^*, \quad \text{fixed by technology;}\\
&K^*/Q = v, \quad \text{variable, according to equation (66)}\\
&x := K/K^* = v^*/v, \quad \text{variable with changes in demand.}
\end{aligned}$$

Assume thus $\eta_1 > 0$ and $\eta_2 > 0$; the reference system becomes:

$$\frac{\dot{\theta}}{\theta} = \beta_1 \left[\left(\tfrac{1-\epsilon}{u_0} \right)^{-\rho} - 1 \right] + \beta_3 p^e - p + \beta_4 - (1-\beta_5)\rho \tag{83a}$$

$$\frac{\dot{\epsilon}}{\epsilon} = \tfrac{1+\eta_2}{1+\eta_2-\hat{\eta}_1} \left\{ (1+\mu) \left[\tfrac{s_p-(s_p-s_w)\theta}{v^*\epsilon^{-\eta_1}(1-\theta)^{\eta_2}} - \gamma - \delta - n(t) \right] + \tfrac{\hat{\eta}_2}{1+\eta_2} \tfrac{\dot{\theta}}{1-\theta} - \rho \right\}. \tag{83b}$$

By simplicity consider at first the case in which $\eta_1 > 0$ and $\eta_2 = 0$. Thus equation (83b) reduces to:

$$\frac{\dot{\epsilon}}{\epsilon} = \frac{1}{1-\hat{\eta}_1} \left\{ (1+\mu) \left[\frac{s_p - (s_p - s_w)\theta}{v^*\epsilon^{-\eta_1}} - \gamma - \delta - n(t) \right] - \rho \right\}. \tag{83b$'$}$$

The Jacobian matrix of system (83a)-(83b′) evaluated at the equilibrium and the corresponding characteristic equation are, respectively:

$$\mathbf{J}^* = \begin{bmatrix} 0 & B \\ -C & \eta_1 D \end{bmatrix}, \quad \text{and} \quad \lambda^2 - \eta_1 D\lambda + BC = 0,$$

where

$$\begin{aligned}
A &:= \theta^*\beta_2(1+\mu)(s_p - s_w)/v > 0;\\
B &:= \theta^*\beta_1\rho[(1-\epsilon^*)/u_0]^{-\rho-1}/u_0 > 0;\\
C &:= \epsilon^*(1+\mu)(s_p - s_w)/[v(1-\hat{\eta}_1)] > 0;\\
D &:= (1+\mu)\sigma/[v(1-\hat{\eta}_1)] > 0.
\end{aligned}$$

It is possible to prove that for the eigenvalues of the Jacobian matrix to be negative it is necessary that $\eta_1 < 0$; but this contrasts with our assumptions. At the roots of this instability result there is the following mechanism: a high unemployment rate entails an increase of the K^*/Q ratio (see eqn. (66)) that decreases the rate of capital utilization $x = v^*/(K^*/Q)$; this reduces profits, $\Pi = x \cdot (PK^*/v^*) - W$. On its turn this reduces savings (see eqn. (69)) and investments through eqn. (70); the demand for employment drops and the unemployment rate rises further.

4.2 Structural change

Another heterodox approach which permits us to investigate our topic in a very peculiar and enlightening way is that put foreword by Pasinetti in its structural change analysis. His framework has been conceived to give a general representation of the dynamics of modern industrial systems. The first formulation of the framework has been provided in Pasinetti (1965); a more general version is presented in Pasinetti (1981). To our purposes it is enough to refer to the simplified version of the model presented in Pasinetti (1993): this latter is based on the assumption that all productive processes require only labour as input; this is, on one side, an expositional device, but, as Pasinetti maintains, it permits to catch "the really fundamental core of a whole family of models, and basically of that whole stream of economic thought that, starting with classical economics, was in this century resumed by Keynesian and post-Keynesian economic theory." Pasinetti (1993, p. xiv).

4.2.1 The basic model: a pure-labour economics

Consider a technologically advanced economy in which M commodities are produced employing labour only. Each individual produces a particular commodity only; due to this marked division of labour each individual can achieve very high levels of productivity, but he will have to obtain all other commodities he needs through exchange. We represent this economy by means of a closed Leontief system; we have, therefore, two linear systems, one for quantities and the other one for prices; given the assumptions on technology the coefficient matrix assumes the following simplified form:

$$\begin{bmatrix} 1 & \dots & 0 & \dots & 0 & -c_1 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & \dots & 0 & -c_m \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & \dots & 0 & -c_M \\ -l_1 & \dots & -l_m & \dots & -l_M & 1, \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_m \\ \vdots \\ q_M \\ q_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{84}$$

$$\begin{bmatrix} 1 & \dots & 0 & \dots & 0 & -l_1 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & \dots & 0 & -l_m \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & \dots & 0 & -l_M \\ -c_1 & \dots & -c_m & \dots & -c_M & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ \vdots \\ p_m \\ \vdots \\ p_M \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \tag{85}$$

where c_m and l_m are, respectively, the *per capita* consumption and the labour coefficient of commodity m, and q_m and p_m are, respectively, the quantity produced and the price of commodity m, $m = 1, \dots, M$. The necessary and sufficient condition for systems (84) and (85) have a non-trivial solution is that the determinant of the two matrices is equal to zero; this amounts to impose:

$$\sum_{m=1}^{M} c_m l_m = 1. \tag{86}$$

This condition, when satisfied, introduces a degree of freedom for both the quantity system and the price system. Their solutions are:

$$q_m = c_m q_N, \quad m = 1, \dots, M \tag{87}$$

and

$$p_m = w l_m, \quad m = 1, \dots, M. \tag{88}$$

To eliminate the two degrees of freedom of the solution we have to chose one quantity and one price from outside. For the quantity system it seems reasonable to fix the quantity of labour force as given from outside, i.e. to set

$$q_N = N.$$

For the price system we can chose any commodity (single or composite) or the labour unit as numeraire of the price system, i.e. to set

$$p_\mu = 1, \quad \text{or} \quad \sum_{m=1}^{M} b_m p_m = 1, \quad \text{or} \quad w = 1.$$

A crucial element of this framework is condition (86). Observe, first of all, that even in this system, where all industrial interdependences have been ruled out at the beginning, this condition links all sectors at the same time: it reflects the interdependence among all sectors due to necessity that each worker has to address to other sectors to get the final commodities he needs. The fact that condition (86) refers to the economic system in its entirety, confers to this relation the character of an actually *macro-economic* condition. To fully appreciate its economic meaning, notice that it can be written in two alternative manners; in fact, thanks to (87) the generic addendum of (86) may be written as $c_m l_m = l_m q_m / N$: it represents the *proportion* of employment required by the productive process of commodity m. On the other hand thanks to (88) the generic addendum of (86) may be written as $c_m l_m = p_m c_m / w$: it represents the *proportion* of potential income generated in each sector m-th by the expenditure addressed towards that sector by the effective demand of consumers. By taking into account of this twofold interpretation of the elements of the sum on the l.h.s. of (86) we can rewrite the macroeconomic condition as:

$$\sum_{m=1}^{M} l_m q_m = q_N \quad \text{and} \quad \sum_{m=1}^{M} p_m c_m = w.$$

It is immediate to see that, in this case, when (86) is not satisfied, it may happen that:

$$\sum_{m=1}^{M} c_m l_m \lesseqgtr 1 \quad \Leftrightarrow \quad \sum_{m=1}^{M} l_m q_m \lesseqgtr q_N \quad \Leftrightarrow \quad \sum_{m=1}^{M} p_m c_m \lesseqgtr w.$$

Then when in macroeconomic condition (86) prevails the symbol '$<$', labour requirements are *less than* the existing labour force—i.e. we have unemployment—and the expenditure for consumption is *less than* income for each worker. On the contrary, when in (86) prevails the symbol '$>$' labour requirements are *higher than* labour force and the expenditure for consumption is *higher than* income for each worker. Thus in order to achieve full employment it is necessary that the expenditure of national income is complete, so that effective demand exerts itself at that level which generates a production equal to the entire potential income of the economic system.

All this for what concerns the *static* version of the model. But there is no reason to suppose that the set of data of this model, i.e. population, N, labour coefficient, l_m, and consumption coefficients, c_m remain constant as time goes on.

4.2.2 Introduction of dynamics

We may suppose, in analogy to what has been done before, that population grows at a proportional rate, n, i.e.,

$$N(t) = N(0)e^{nt}. \tag{89}$$

We suppose, moreover, that labour productivity increases; in terms of our model this phenomenon takes the form of a decrease of each labour coefficient, i.e.

$$l_m(t) = l_m(0)e^{-\rho_m t}, \quad m = 1, \ldots, M. \tag{90}$$

Suppose, by the end, that per capita consumption increases, i.e.

$$c_m(t) = c_m(0)e^{r_m t}, \quad m = 1, \ldots, M. \tag{91}$$

Substitute (89), (90) and (91) into the equations of the model, (84) and (85). By solving it we obtain a *path* for the quantities produced of each commodity:

$$q_m(t) = c_m(t)N(t) = c_m(0)N(0)e^{(r_m+n)t}, \quad m = 1, \ldots, M. \tag{92}$$

Solution (92) emphasizes a *structural change of quantities*: the quantity produced of each commodity change at a rate which is the sum of the rate of increase of population, n, and the rate of increase of demand of commodity m, r_m; while n is uniform across commodity, r_m is in general different; thus the total rate at which the production of each commodity evolves is different, in general, from commodity to commodity.

On the other side, if we consider the prices of the various commodities, expressed in terms of a commodity μ, chosen as numeraire, we obtain:

$$\frac{p_m(t)}{p_\mu(t)} = \frac{w(t)l_m(t)}{w(t)l_\mu(t)} = \frac{l_m(0)}{l_\mu(0)}e^{-(\rho_m-\rho_\mu)t}, \quad m = 1, \ldots, M. \tag{93}$$

Equations (93) entails a *structural dynamics of prices*: the prices of the commodities whose productivity grows faster than that of the industry that produces the numeraire commodity will decrease, and vice versa the other ones.

4.2.3 Structural change of employment

The introduction of dynamics entails also a series of consequences involving employment. Given the multisectoral nature of the this framework we can study the problem either at the sectoral level or at the macroeconomic level. Let us begin to observe the phenomenon at a *sectoral* level: to see this suppose that at the macroeconomic level condition (86) is kept satisfied.

At *sectoral* level we can observe that the *proportion* of employment required in each sector m, changes continuously according to equation:

$$\frac{l_m(t)q_m(t)}{N(t)} = c_m(t)l_m(t) = c_m(0)l_m(0) \cdot e^{(r_m-\rho_m)t}, \quad m = 1, \ldots, M: \tag{94}$$

the share of total employment employed in sector m increases (decreases) if the rate of change of per-capita demand of commodity m, r_m, is higher (lower) than the rate of change of labour productivity, ρ_m. Thus the structural change of technical coefficients and of demand coefficients requires a continuous *intersectoral mobility* of labour. This phenomenon is less pronounced if we observe the *absolute level* of employment required in each sector m,

$$E_m(t) := l_m(t)q_m(t) = l_m(0)c_m(0)N(0) \cdot e^{(n+r_m-\rho_m)t} \quad m = 1, \ldots, M: \tag{95}$$

the level of employment in sector m increases (decreases) if the rate of change of *total* demand of commodity m, $n + r_m$ is higher (lower) than the rate of change of labour productivity, ρ_m. We see thus a typical phenomenon of growing processes generated by technical progress: the presence of sectors in contraction alongside that of sectors in expansion. According to Pasinetti this is "one of the most alarming phenomena of modern industrial systems: the inevitable decline of employment in certain production sectors, as a result of the process of economic development." Pasinetti (1993, p. 53).

4.2.4 Technological unemployment

In the previous subsection (4.2.3) we supposed that the macroeconomic condition (86) were satisfied. But this cannot be taken for granted at all. On the contrary the main effect of technical change on (86) is to reduce the coefficients l_m and, by consequence, the addenda of the sum on its l.h.s. Hence consumption coefficients have to increase in order to keep condition (86) satisfied. Thus we can see that the increase of consumption is not only a *possibility*, but it becomes a *necessity* in order to preserve the full employment equilibrium. But we could go further. In fact, notwithstanding our simple formulation of the evolution of consumption (see eq. (91)), coefficients c_m cannot be thought to increase indefinitely: since the times of Engel it is known that when income increases the consumption of the various commodity increases within a certain interval of income (except for inferior commodities, whose consumption decreases with respect to income), but after a certain level we observe a phenomenon of saturation. Further increases of income will be spent on *other* commodities. It is possible to see that there is a sort of hierarchy of consumptions: low income levels are spent to purchase primary goods, higher income levels are spent in higher quality goods—decreasing thus the consumption of some 'inferior' goods); again, further increases of income are spent to buy some luxury goods and so on. All this permit us to think that coefficients c_m do not increase at a constant rate, as supposed in (91). This entails that the movements of the c_m sooner or later will not be sufficient to contrast the decreases of labour coefficients; thus for condition (86) there is a tendency to become satisfied with the '$<$' symbol. Hence according to this framework a growing process characterized by structural change tends to generate unemployment.

It could be argued that even labour coefficients could decrease at non-constant rates, like supposed for consumption coefficients. In effect Pasinetti reformulates the whole model to allow for non-constant rates of change of both l_m and c_m coefficients.[22] But it is not unrealistic to imagine that the forces that drive the reduction of the rates of change of consumption coefficients work harder than the forces that reduce the increases of labour productivity. In any case what we can learn from this framework is that even if we start from a situation with full employment of labour, this situation needs not to continue to be satisfied. The macroeconomic condition (86) shows us which are the forces that *may* induce unemployment. But the same framework shows us which are the forces that can act and which remedies can be introduced to fight unemployment. To see this it is convenient to reformulate the model by removing the simplifying assumption of coincidence between population, $N(t)$, and working population, $q_N(t)$. It is reasonable to suppose that among $N(t)$ there is a fraction $\mu(t)$ of working-population and its complement, $1-\mu(t)$, that does not contribute to productive activity, even if it contributes to consumption. Similarly we could suppose that of the full time available to each individual a fraction $\nu(t)$ is devoted to work, and its complement, $1-\nu(t)$, devoted to leisure. Hence the quantity of available

[22] See Pasinetti (1993, pp. 77–79).

labour is $q_N(t) = \mu(t)\nu(t)N(t)$. Thus model (84)-(85) is to be reformulated accordingly,[23] and the correspondent macroeconomic condition would become:

$$\frac{1}{\mu(t)\nu(t)} \cdot \sum_{m=1}^{M} c_m(0) l_m(0) \cdot e^{(r_m - \rho_m)t} = 1. \tag{86$'$}$$

We can now return to the forces and the remedies against unemployment. From (86′) it emerges that the first way to contrast unemployment is the increase of individual consumption. It has been said that this force fails to work in the long-run. At this point we could hope that the increases in demand involve the commodities produced in those sectors where the productivity increases are lower. Exportations as well could help an economy in opening new markets for the products, but we should have to consider, at the same time, the opposite effects of importations.

Another way to contrast unemployment, that has greatly took place in modern economies, has been the increase in the *number* of new commodities. This might be easily inserted into the present framework by supposing that the number of commodities is a function of time, $M(t)$. Thus even each addendum of (86′) decreases its sum can keep equal to 1 as the number of addenda increases.

Another way to avoid (or to reduce) unemployment could be the reduction of parameters $\mu(t)$ or $\nu(t)$, i.e. an increase of the number of part-time workers, or of retirement, or a reduction of the working time. It should be said that all these elements are not alternative one another; on the contrary they can introduced together, and the task of institutions is really that of choosing the appropriate mix.

4.2.5 Proportional dynamics

The present framework includes, as a particular one, the case of proportional dynamics, that has extensively studied in the literature (see the main part of input-output dynamic analysis or the main part of long-run aggregative models). It is obtained by setting

$$\rho_m = \rho, \quad 1, \ldots, M,$$

[23] System (84) becomes:

$$q_m(t) = c_m(t)N(t) = \frac{c_m(t)}{\mu(t)\nu(t)} \cdot q_N(t), \quad m = 1, \ldots, M$$
$$\sum_{m=1}^{M} l_m(t) q_m(t) = q_N.$$

System (85) becomes:

$$p_m(t) = w(t) l_m(t) \quad m = 1, \ldots, M$$
$$\sum_{m=1}^{M} p_m(t) c_m(t) N(t) = \sum_{m=1}^{M} p_m(t) \frac{c_m(t)}{\mu(t)\nu(t)} \cdot q_N(t) = q_N.$$

i.e. by supposing a *uniform labour augmenting technical progress*, like that used in the the main part of growth models (and in all models previously presented in this survey, with the exception of the Aghion-Howitt model). Normally in these analysis it is supposed that also all consumption coefficients increase at the same uniform rate, which has to be equal to the rate of increase of productivity, i.e.

$$r_m = r = \rho, \quad m = 1, \ldots, M.$$

In this case we have an homotetic growth of the system: from (92) we see obtain that all quantities produced increase at the same rate, $q_m(t) = c_m(0)N(0)e^{(n+r)t}$, $m = 1, \ldots, M$; from (93) we find that all relative prices remain constant, $p_m(t)/p_\mu(t) = l_m(0)/l_\mu(0)$, $m = 1, \ldots, M$. Moreover as $r_m - \rho_m = 0$ for $m = 1, \ldots, M$ the macroeconomic condition simplifies to

$$\frac{1}{\mu(t)\nu(t)} \cdot \sum_{m=1}^{M} c_m(0)l_m(0)\cdot = 1;$$

thus if it is satisfied for $t = 0$ it continues to be satisfied forever. By observing that a quite wide number of models of growth are based upon this case of dynamics it emerges one more reason why the literature on growth has ever found so difficult to study the unemployment problems within growth theories.

5 Concluding remarks

In this review we have analyzed several works belonging to very different approaches to growth theory, each giving a different focusing on the links between growth and unemployment. For each of these frameworks we referred to the simplest version, in order to catch clearly the basic elements from which conclusions are drawn.

We begun with the Harrod-Domar model, (Harrod, (1939), Domar, (1946)), whose assumptions bring to obtain an unstable long-run equilibrium path: unless, by a fluke, the growth rate that guarantees a full employment of productive capacity (warranted rate of growth) coincides with the growth rate of population (natural rate of growth), *the system experiments an ever increasing unemployment or ever increasing inflation pressures.* Then we considered the Solow (1956) model, which was conceived to provide a solution to the Harrodian instability paradox. In the Solow's model *unemployment is completely ruled out*: instantaneous movements of factor prices clear the factors markets, inducing changes in the capital/labour ratio that adjust the warranted rate to the natural rate of growth.

This solution conditioned hardly the subsequent development of growth theory for about thirty years—perhaps besides Solow's own intents—rendering very complex to study any form of unemployment within standard growth theory.

The search theory of unemployment, recently developed by Pissarides (1990) and (2000), is an attempt to overcome this limit. This theory explains unemployment as an 'equilibrium' phenomenon, due to frictions in the labour market; but while this explanation can be accepted if referred to a short-run horizon it appears not convincing when inserted in a long-run framework. The peculiar result obtained by Pissarides for growing economies is the following effect, known as 'capitalization effect': if the rate of change of productivity, ρ, increases, firms find more convenient to anticipate hiring, as hiring costs increase with ρ; thus *an increase of ρ decreases the unemployment rate*. But this effect does not seem very relevant to explain any real phenomenon.

The most convincing Neoclassical explanation of unemployment in a growth context has been provided by neo-Schumpeterian models developed by Aghion and Howitt. In Aghion and Howitt (1994) the creative-destruction of jobs induced by technical progress is analyzed very clearly: according to their 'creative-destruction' effect *an increase of ρ increases the unemployment rate*. They, however, underestimate the problem of vents (*débouchés*) for the production, but the notion of effective demand notoriously hardly fits in Neoclassical frameworks.

On the contrary this latter is a notion that can be dealt more satisfactorily in non-orthodox theories. In the present work we saw two non-orthodox growth theories which give different insights of the links between growth and unemployment. The first one is that based on the works of Goodwin (1967) and of Akerlof and Stiglitz (1969). Their models are based on a quite (realistic) conflictual vision of the relationships between capitalists and workers: unemployment decreases real wages; this stimulates capitalists' investments; these latter, by raising labour demand, raise real wages, and this counterbalance the initial push, investments decrease, and so on. All this yields a *cyclical dynamics for the unemployment rate and the wage share*.

The role for effective demand emerges distinctly from the other non-orthodox model here presented: the structural dynamics analysis put foreword by Pasinetti. In it the 'creative-destruction' effects of technical progress are directly faced with the absorptions capabilities of final demand: *the interplay of technical progress and increases in consumptions may result in an underemployment equilibrium*, and the model is immediately suitable to envisage the possible remedies to this disequilibrium situation. The results of the model are in some respect indeterminate, as the author decided to leave some (maybe too much) variables as exogenous (in particular the rates of increase of productivity and of per-capita demand). An integration of the neo-Schumpeterian hints with a treatment along Keynesian lines of the demand side of an economy seems to be a promising step to analyze the interactions between growth and unemployment.

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