Job Contact Networks, Inequality and Aggregate Output

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Abstract

In this paper we study the effects of social networks on wage inequality and aggregate production. In particular, we consider a simplified version of the model by Calvó-Armengol and Jackson (2003), with good and bad jobs and skilled and unskilled workers. Our findings are: i) increasing the number of social links increases aggregate output and may reduce inequality; ii) given a number of social connections, output increases if the average distance among worker decreases; iii) a more mixed and well-integrated society, that is a society in which heterogeneous workers share social links, produces more output and less inequality than a society in which some workers are isolated, when productivity of the most productive agents in the best jobs is sufficiently low. We draw some policy implications from these results.

Keywords: Social Networks, Wage Inequality, Aggregate Output

JEL Codes: A14, J31, J38

1 Introduction

The importance of social networks in labor markets is well-documented in the sociological literature (e.g. Granovetter (1974)) which highlights the importance of social links, like friends, relatives and acquaintances, as sources of information on jobs. A number of empirical studies report that approximately between 40% and 60% of employed workers found their jobs through social networks although, in general, these proportions vary with sex, occupations, skills, and workers' socio-economic background.[1]

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[1]See Montgomery (1991) for further discussion and references. Other works on this field are Holzer (1987), Green et al. (1999) and Topa (2001). Pistaferri (1999) is a study on Italian data.

Another line of empirical research shows that observable individual characteristics (e.g. education, skill level, abilities, family, etc.) account for only about 50% of wage inequality (see Arrow and Borzekowski (2003) for references). The fact that workers have different social ties or links can play a role in explaining such an evidence. In particular, all other variables held constant, workers with different networks will have on the average different wages and employment opportunities. Furthermore, as remarked by Calvó-Armengol and Jackson (2004), variables such as workers' location or race may capture network effects, and therefore they can interact with other workers' individual characteristics in explaining wage outcomes and inequality.

Joining a small but growing economic literature, we model social networks in labor markets in order to investigate their role in explaining wage inequality among workers, as well as aggregate production. In particular, we consider a simplified version of the model by Calvó-Armengol and Jackson (2003), in which information about heterogeneous jobs arrives randomly to heterogeneous agents. We study the case of two types of jobs (good/bad) and two types of workers (skilled/unskilled). Unemployed workers accept any offer while employed workers accept it only if the job is more attractive (in terms of pay) of the current one. If this is not the case, they pass the information about the vacancy to a worker in their network.

We find that, in general, the geometry of the network affects aggregate production and inequality. In particular, we show that: i) increasing the number of links in a network increases output and reduces inequality; ii) for a given number of social links connecting all agents, output increases if the average distance among workers decreases; iii) for a given number of social links, output increases and inequality decreases when all agents have some links, given that the productivity of skilled workers is sufficiently low.

The rest of the paper is structured as follows: in Section 2, we offer a brief overview of the related literature; in Section 3, the basic model is introduced and described; in Section 4, we present and analyze some simple examples; in Section 5, results of simulations are reported and discussed; in Section 6 we derive some policy implications; Section 7 concludes.

2 Related literature

A fundamental contribution in economics on the role of social networks in labor markets is the seminal work of Montgomery (1991), who presents an adverse selection model in which job referrals improve the quality of firm-worker matches, when firms cannot perfectly observe workers' ability before hiring. In this model, an increase in the density of social ties increases wage inequality. The reason is that social ties convey to firms more information on workers' quality, and this increases the gap between the (higher) wage paid to referred workers, and the market wage paid to those who find a job through other channels.

Montgomery (1994) analyzes also the role of "weak ties", that is relationships with non frequent social interactions (or transitory relations), and shows that they are positively related to the aggregate employment rate. Furthermore, weak ties reduce inequality, measured by the distribution of employment which obtains with social interactions, relative to a case of absence of a social network, in which individuals are randomly allocated to jobs.

In our model, inequality does not depend on adverse selection,[2] but on the network structure. Furthermore, differently from Montgomery (1994), we do not consider inequality only in terms of employment opportunities but also in terms of wage differentials.

Arrow and Borzekowski (2003) propose a static model which focuses on wage inequality determined by differences in the number of connections of workers to firms, in an imperfect information framework where firms have more information on workers connected to them. In this environment, workers with different number of connections have on average different incomes. In particular, they find that about 13-15% of the variation in log wages is attributable to the variation in the number of workers' connections.

Firms are imperfectly informed on workers' productivity also in the dynamic model of Krauth (2004). In this model employed workers may provide information on the skills of their unemployed friends, and the number of connections is positively related to employment (both for individual workers and in the aggregate).[3]

In our framework, the mechanism through which social networks affect employment, productivity and wages in the economy is quite different from that emphasized in Arrow and Borzekowski (2003) and Krauth (2004). In particular, here the social network is the channel by which workers increase their probability to find a (better) job, rather than the channel by which firms acquire more information on workers' productivity.

Our paper closely follows Calvó-Armengol and Jackson (2003), who present a very general model with exogenous networks[4] among workers, which facilitate the transmission of information on job vacancies. They show that both wages and employment are positively correlated across time and workers. Furthermore, differences in drop-out rates from the labor force are explained by the different social networks of workers. In a companion paper, Calvó-Armengol and Jackson (2004) analyze in more detail the spe-

[2]Another paper that study the effects of social networks on inequality in an adverse selection framework is Finneran and Kelly (2003).

[3] Krauth (2004) also shows that average employment is positively related to the fraction of weak ties for a given number of connections.

[4]For models in which the formation of the network is endogenous, see Jackson and Wolinsky (1996), Bala and Goyal (2000) and Calvó-Armengol (2004).

cial case with identical jobs and a single wage level, providing also a number of simulation results. In our model with heterogeneous jobs and workers, we extend their framework to the study of the dynamics of aggregate output and inequality, as well as their correlation.

3 A model of labor market with social networks

3.1 Production, wages and turnover

We present a model of labor market which derives from Calvó-Armengol and Jackson (2003). In particular we study the case with two types of jobs and two types of workers. Time is discrete and indexed by $t = 0, 1, 2...$ The economy is populated by a number of risk-neutral, infinitely-lived agents (workers) indexed by $i \in \{1, 2, ..., N\}$. In each period a worker can be either employed or unemployed. Indicating with the variable θ the employment status of the worker, $\theta \in \{0 \text{ (unemployed)}, 1 \text{ (employed in job 1)}, 2 \text{ (employed in job 2)}\}$ and with the variable λ her or his type, $\lambda \in \{1, 2\}$, we have that each agent in every period can be in one of the following states:

$$s_{it}^{\lambda\theta} = \begin{cases} s^{11} \text{ if type 1 and employed in job 1} \\ s^{12} \text{ if type 1 and employed in job 2} \\ s^{22} \text{ if type 2 and employed in job 2} \\ s^{21} \text{ if type 2 and employed in job 1} \\ s^{10} \text{ if type 1 and unemployed} \\ s^{20} \text{ if type 2 and unemployed} \end{cases}$$

On the production side, we consider one-to-one employment relationships (that is each firm need a single worker to produce), and assume a very simple form of a production function, in which productivity depends on the type of match between the worker and the job (firm). In particular, we denote with $y_{it}^{\lambda\theta}$ the output of a firm employing worker i, at time t, for a match $\lambda\theta$ or, in other words, the surplus generated by match $\lambda\theta$ (output price is normalized to one).

In this paper, we focus on an hi/low skill and good/bad job economy. First, we assume that worker 2 is more productive than worker 1, for instance because s/he is more skilled. Second, we consider job 2 as being more productive than job 1, for instance because it is a hi-tech, good job. According to these assumptions, the parameter $y^{\lambda\theta}$, indexing the productivity of a match, follows the rule:

$$y^{22} > y^{12} = y^{21} > y^{11} > 0 (= y^{10} = y^{20}).$$

In other words we assume that the highest (lowest) productivity obtains when a skilled (unskilled) worker has a good (bad) job.[5] Other cases fall in between, and for simplicity are assumed to give the same product[6].

Wages are a fraction of match surplus, and are denoted by $w^{\lambda\theta} = \beta y^{\lambda\theta}$ with $\beta \in (0, 1)$.[7] This produces an ordering of wages obtainable in a given match, which follows the ordering of outputs. Obviously, unemployed workers earn zero wages, and we assume that their reservation utility is zero.

The labor market is subject to the following turnover. Initially, all workers are unemployed. Every period (from $t = 0$ onwards) has two phases: at the beginning of the period each worker receives an offer of a job of type f, with $f \in \{1, 2\}$, with arrival probability $a_f \in [0, 1]$. If the agent is already employed, and not interested in the offer in the sense that the offered job has a lower wage, s/he passes the information to a friend/relative/acquaintance who is unemployed or employed but receiving a lower wage then the one paid for the offered job. At the end of the period every worker loses the job with breakdown probability $b \in [0, 1]$.

3.2 Social links and job information transmission in a hi/low skill - good/bad job economy

Social networks in the economy may be conveniently represented by a graph g which summarizes the links of all agents, where $g_{ij} = 1$ if i and j know each other, and $g_{ij} = 0$ indicates that they do not know each other. It is assumed that $g_{ij} = g_{ji}$, meaning that the acquaintance relationship is reciprocal. Given the assumptions on wages and arrival probabilities, the probability of the joint event that agent i learns about a job and this job ends up in agent's j hands, is described by $p_{ij}(s_{it}^{\lambda\theta})$:

$$p_{ij}(s_{it}^{\lambda\theta}) = \begin{cases} a_f \text{ if } j = i \text{ and } s_i = s^{\lambda 0} \text{ or } s_i = s^{\lambda 1}, f \neq \theta \\ a_f \frac{g_{ij}}{\sum_{k:s_k = s^{\lambda 0}} g_{ik}} \text{ if } f = 1, s_i = s^{\lambda\theta(\theta \neq 0)}, s_j = s^{\lambda 0} \\ a_f \frac{g_{ij}}{\sum_{k:s_k = s^{\lambda\theta(\theta \neq 2)}} g_{ik}} \text{ if } f = 2, s_i = s^{\lambda 2}, s_j = s^{\lambda\theta(\theta \neq 2)} \end{cases}$$

In the first case, worker i receives with probability a_f and takes for her/himself a job offer. This holds if s/he is either unemployed or employed in a bad job when s/he receives an offer for a good job. In the second case the worker i is employed and receives with probability a_1 an offer for a bad

[5]In a work in progress we consider also a good/bad match economy, in which the highest productivity is obtained in all matches of the type $\lambda\theta$ with $\lambda = \theta$, and compare (in terms of wage inequality and aggregate output driven by the structure of social networks) results obtained for the two "economies".

[6]Notice that we are assuming that skills have a certain degree of transferability across jobs, since y^{12} and y^{21} are strictly positive. Putting it another words, skills are partially general (see Becker (1964) for the distinction between general and specific skills).

[7]For instance β may represent the bargaining power of workers when wages are set by Nash bargaining, as is usual in search models. Clearly, profits are $(1 - \beta)y^{\lambda\theta}$.

job, that s/he passes only to an unemployed worker $j(\neq i)$. We assume that among all unemployed workers connected with i by a social link, i chooses j randomly. Hence, the probability that worker j receives the information by worker i is equal to $\frac{g_{ij}}{\sum_{k:s_k=s^{\lambda 0}} g_{ik}}$. In the third case the worker i is employed in a good job and receives an offer for a good job with probability a_2, thus s/he passes the offer to a worker connected with her/him who is employed in a bad job or unemployed, with probability $\frac{g_{ij}}{\sum_{k:s_k=s^{\lambda\theta(\theta\neq 2)}} g_{ik}}$.

To sum up, a worker who receives an offer makes direct use of it if the new job opportunity increases her/his wage. Otherwise, s/he passes the information to someone who is connected with her/his. The choice of the worker to whom pass the information is "selective", in the sense that the information is never passed to someone who does not need it,[8] but it is random with respect to the subset of the connected workers who improve their condition (wage) exploiting such an information (for example, a worker receiving a good job offer is indifferent to pass it to an unemployed contact or a contact employed in a bad job[9]). Finally, we exclude that job information may be transmitted to more than one (connected) worker.[10]

4 Some simple examples

We begin by presenting some examples. First, we consider the role of social ties on the expected wage of a worker. Second, we illustrate the potential effects of changing the network geometry on wage inequality and aggregate output. Although these examples are very simple, they are useful to introduce the effects of social networks on wages and output, as well as other relevant aspects that we will investigate afterward in more detail by providing a number of simulations results and discussing policy implications.

4.1 Social links and expected wages

Consider an unemployed worker i in period t, that is a worker who entered the period unemployed and did not receive any offer in that period. Her/his state at the end of period t is $s^{\lambda 0}$. Her or his expected wage in period $t+1$, when the expectation is formulated in period t, is strictly dependent on the network she or he belongs, which is the same in all periods. As examples, we consider now two possible situations. In the first example, the worker has no social ties; in the second example, agents i and j are connected, j is employed in a bad job in period t and has no other links.

[8] If all of the worker's acquaintances do not need the job information, then it is simply lost.

[9] Hence, these agents are "competitors" for the information on such a vacancy (see below).

[10] Calvó-Armengol and Jackson (2003) provide various extensions on the process of transmission of job information.

Example 1 Figure 1 represents the case in which worker i has no links.

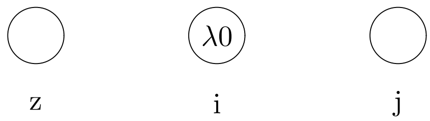


Figure 1: No links

In this case, the expected wage in $t+1$ for worker i depends only on the exogenous probabilities that s/he directly receives some job offer at the beginning of that period. In this case, since worker i accepts an offer for a bad job only if she or he does not receive an offer for a good job, her or his expected wage in the next period is equal to:

$$Ew_{i,t+1} = a_2 w^{\lambda 2} + a_1(1-a_2)w^{\lambda 1}$$

Example 2 Figure 2 represents the case in which worker i has a link with worker j (who has no links other than with i).

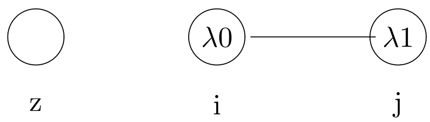


Figure 2: One link with a worker in a bad job

Now worker i can also find a job, other when directly receives some offer, when worker j passes to her or him some job information. Of course, being employed in a bad job, worker j passes to worker i only information about a bad job and retains for herself or himself an offer for a good job. In particular, worker j passes an offer for a bad job to worker i if the former does not lose the job (with probability $(1-b)$) at the end of period t, and receives an offer for such a job (with probability a_1) at the beginning of time $t+1$, or if s/he loses the job (with probability b) at the end of time t and receives both an offer for a good and a bad job (with probability $a_1 a_2$) at the beginning of time $t+1$. Thus, in this context, worker i's expected wage in $t+1$ is given by

$$
\begin{aligned}
Ew_{i,t+1} = {} & \underbrace{a_2 w^{\lambda 2}}_{i \text{ receives an offer for job 2}} + \underbrace{a_1(1-a_2)w^{\lambda 1}}_{i \text{ receives an offer for job 1 and not for job 2}} + \\
& + \underbrace{a_1(1-b)(1-a_2)(1-a_1)w^{\lambda 1}}_{j \text{ does not lose the job and receives an offer for job 1; } i \text{ exploits it if s/he receives no offers}} + \\
& + \underbrace{a_1 a_2 b(1-a_2)(1-a_1)w^{\lambda 1}}_{j \text{ loses the job and receives both an offer for job 1 and 2; } i \text{ exploits the offer for job 1 if s/he receives no offers}} = \\
& = a_2 w^{\lambda 2} + a_1(1-a_2)[1 + (1-b)(1-a_1) + a_2 b(1-a_1)]w^{\lambda 1}.
\end{aligned}
$$

Since the expression in square brackets is greater than 1, the expected wage for the worker i in $t+1$ is now higher than in Example 1 i.e. the social link with worker j has a strictly positive effect (on average) for worker i.

Obviously, when an unemployed worker is linked to a worker employed in a good job, the latter may pass offers for both types of jobs (conditioned on keeping her/his job at the end of period t). This increases the expected wage in period $t+1$ for worker i.[11]

Example 3 More complicated cases can arise, for instance, when two unemployed workers are "competitors" for information that is when they are both linked to an employed worker who may transmit the information only to one of them. In this case, their wages in period $t+1$ are negatively correlated because they are "competitors". Consider, for example, Figure 3 in which, at time t, worker i has a link with worker j who has in turn a link with another unemployed worker, worker z).

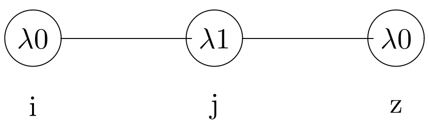


Figure 3: One link with a worker in a bad job with another link

In such a case, worker j passes only with probability an half a (bad) job offer to worker i. For such a reason, worker i expected wage in $t+1$ is equal to:

$$Ew_{i,t+1} = a_2 w^{\lambda 2} + a_1(1-a_2)[1 + \frac{(1-b)(1-a_1) + a_2 b(1-a_1)}{2}]w^{\lambda 1}.$$

[11] This aspects are fully analyzed in Calvó-Armengol and Jackson (2003). They show that increasing the wage of any of an agent's connections leads to an increase (in the sense of stochastic dominance) in the probability that the agent will be employed and the agent's expected wages.

Since in this case the expression in square brackets is lower than the corresponding one in Example 2, the expected wage for the worker i in $t+1$ is now lower than in the previous example. Hence one may conjecture that, *ceteris paribus*, a worker (weakly) prefers to be linked to workers with no other links. However, as stressed by Calvó-Armengol and Jackson (2003), this holds in the short run (that is in a one period perspective), but in a longer run perspective it should be carefully reconsidered. In fact, referring to the mentioned case, the presence of a "competitor", worker z, results useful for worker i since the former helps to improve the wage status of the common connection, worker j, and this, in the longer run, increases the probability that s/he passes more information to worker i. This aspect outweights the local (conditional) negative correlation, due to the "competitive effect", and induce long-run positive correlation between wages of workers i and z (see Calvó-Armengol and Jackson (2003)).

What stated for expected wages holds true for expected outputs, given our assumptions. In Section 5 we focus directly on outputs, while wages are examined in terms of inequality among workers. In particular we study the long-run dynamics by means of simulations.

4.2 Changing the network geometry: inequality and aggregate output

Before presenting the simulations, we describe in a simple form another example which introduces the consequences of a change in the network geometry on inequality and aggregate output, for a given structure of the population and the same size of the network, i.e. the same number of links.

Example 4 Consider the following network structures, g_1 and g_2, and states of four workers in a generic period t:

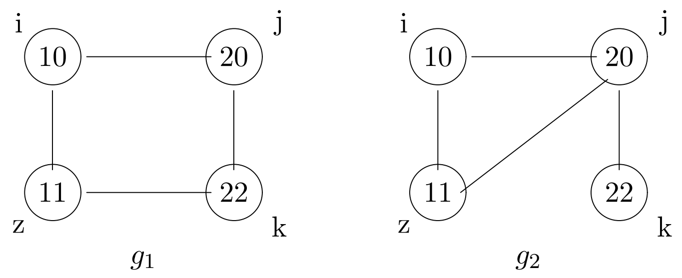


Figure 4: Networks g_1 and g_2

In both cases we have two unemployed workers and two employed workers, in one of the possible jobs, and two workers for each type. Obviously, output, wages and inequality at time t are equal in the two different networks, but they might differ in $t+1$ because of different network geometry. For both networks we compute the following expected values for period $t+1$: i) average output; ii) wages for each worker; iii) wage inequality, measured by the Gini index of (expected) wages .[12]

Network	Output	Wages[i,j,z,k]	Inequality
g_1	2.632	0.55,1.31,0.75,1.60	0.220
g_2	2.655	0.55,1.50,0.60,1.60	0.238

Table 1: Output, wages and inequality

We observe that in network g_2 output has increased by a percentage of 0.8%, as well as inequality, which shows a relatively higher percentage increase of 8%. Output increases as the improvement in the expected output of worker j, which has more links g_2, outweights the worsening of the situation of worker z, which in g_1 has one link less with a worker in a good job. This is reflected in the changes in expected wages, which are simply proportional to expected output in our framework.[13] The change is such to determine an increase in inequality. So, in this simple case, a change in the network structure from g_1 to g_2, simply obtained by a rewiring of only one link, increases output at the price of an increase in inequality.

5 Simulations

In this section we present the results of the simulations.[14] Our aim is to assess how the presence and the structure of social networks affects dynamics of output and wage inequality in the long run. We begin by considering the effects of the number of social ties. Then we explore other aspects of social networks topology, usually considered by the theory of social networks, and finally we study the case in which the number of links is fixed.

[12] Parameters in this simple example are the following: $a_1 = 0.5$, $a_2 = 0.5$, $b = 0$, $y^{11} = 1$, $y^{12} = y^{21} = 2$, $y^{22} = 4$; $\beta = 0.4$. The choice of $b = 0$ is just for simplification and it does not affect the qualitative result that we introduce here.

[13] The fact that the expected wage of worker j increases while that of worker z decreases is an application of Lemma 2 in Calvó-Armengol and Jackson (2003). This states that an agent's probability of being employed, expected number of offers and wages all increase (in the sense of stochastic dominance) if the agent's probability of hearing job information through contacts network improves and *vice versa*.

[14] All simulations was programmed in R (http://www.r-project.org/). The codes of simulations are available upon request from the authors.

5.1 Social links, dynamics and long-run patterns of output and inequality

Consider a network with 4 agents. For simplicity we assume that two workers are unskilled (white dots) and two workers are skilled (black dots), and that initially all workers are unemployed. Hence, at time $t = 0$ we have two workers in state s^{10} and two workers in state s^{20}.[15]

We analyze six possible network configurations (see Figure 5): an empty network g_A, that is a situation in which no social tie exists; a network with one link between unskilled agents (g_B); a network with one link between skilled agents (g_C); a network with one link between agents with different skill levels (g_D).

The last two networks represent more complex "social environments". In particular, g_E is a "path-connected" network, that is a network in which all agents are linked to the two agents on their side, thus with four social ties all agents are (directly and indirectly) connected to each other. Instead, g_F is a complete network in which each agent is directly connected with each other, for a total of six links.

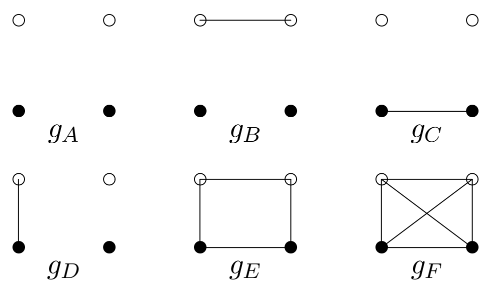


Figure 5: Networks g_A - g_F

g	**Output**	**Inequality**	**Correlation**
g_A	2.751	0.212	-0.773
g_B	2.791	0.197	-0.748
g_C	2.829	0.203	-0.777
g_D	2.810	0.201	-0.785
g_E	2.917	0.181	-0.762
g_F	2.939	0.177	-0.738

Table 2: Output and inequality

We study the relation between the structure of the network and the

[15]Workers are numbered from 1 to 4, starting from top left and counting clockwise.

average production in the network, as well as the degree of inequality. In particular, we simulate the economy for 500,000 periods using these parameters: $y^{11} = 1$; $y^{12} = y^{21} = 2$; $y^{22} = 4$; $a_1 = 0.15$; $a_2 = 0.10$; $b = 0.015$; $\beta = 0.4$.[16] Results of simulations are reported in Table 2. In particular, we present the value of average output per worker, and of average inequality, measured by the Gini index (both averages are computed over the 500000 periods of the simulations). Also, we report the correlation of output and inequality.

Starting from the situation with no social ties, even moving to just a single link ($g_{12} = g_{21} = 1$ in g_B or $g_{34} = g_{43} = 1$ in g_C) we see that (average) output increases and inequality decreases. However, some qualifications are needed according to whether the single link is between low-skill or between hi-skill workers. With respect to g_A, in g_B the increase in output is less pronounced (+1.45% against +2.84% in g_C) while reduction in inequality is stronger (−7.07% in g_B and −4.24% in g_C).

Clearly, these results make sense. Having a link with another worker increases the probability to get a (better) job. This increases the average output during time. Furthermore, since hi-skill workers are more productive when hired in a good job, and having a link increases the probability to get that job, output is greater in g_C. At the same time, since wages are in proportion of output, inequality is greater too (even if it is lower than in the "no links" case) since hi-skill workers, with potential higher wages, are the only agents taking advantage of the social tie.

For reasons that now should be clear, the case of g_D is intermediate between g_B and g_C. In fact, maintaining the same number of social links (one), output and inequality have an intermediate value with respect to g_B and g_C. In this case, therefore, we have the indication of a possible tradeoff: "mixing" the population, that is allowing agents of different type to be connected, decreases output with respect on the case in which two skilled workers are linked, as the flow of good jobs to skilled workers is reduced (in other words more "mismatches" may occur). However, inequality decreases as one unskilled worker has more opportunities to obtain a good job and an higher wage. Of course, the converse holds if we compare the "mixed" situation with one in which two low-skill agents are connected. This result can be extended to networks with more agents as well (see Section 5.3).

As we see from Table 2, adding more links further increases output and decreases inequality. In particular, in a comparison between the two networks in which all agents are connected, we notice that in network g_F output is increased by 6.83% respect to g_A, and inequality is decreased by 16.51%, while in network g_E output has increase by 6.03% and inequality has de-

[16] Values for $a_2 = 0.10$ and $b = 0.015$ and taken from Calvó-Armengol and Jackson (2004), who consider only one type of job. We choose the value of $a_1 = 0.15$ on the assumption that it is more difficult to get a good job than a bad job.

creased by 14.62%. Hence, in this framework an increase in the number of links is unambiguously associated with an increase in average output and to a decrease in inequality.[17]

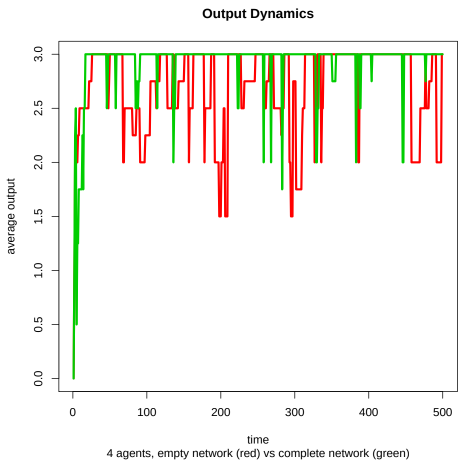
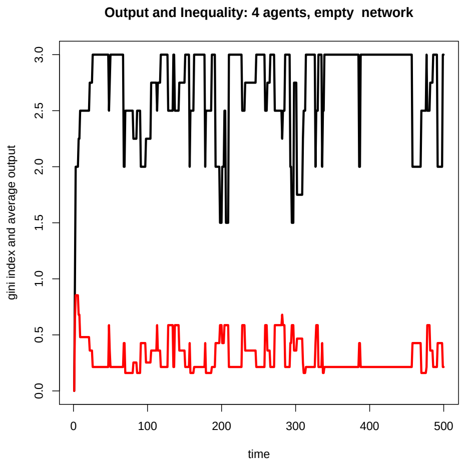


Figure 6: Output dynamics in g_A (red line) and g_F (green line)

In Figure 6 we compare the output dynamics in the two extreme cases: the empty network g_A (red line) and the complete network g_F (green line). As remarked, average output in the complete network is higher, and from the figure we can also observe that it is more stable over time (first five hundred periods of the simulations). Clearly, with all links activated the individual probabilities of being unemployed, and unproductive, are drastically reduced with respect to the empty network.

Output dynamics is compared to inequality dynamics in Figures 7 and 8.

[17]Individual average wages over the period, have a predictable pattern: in g_A unskilled workers' average wage is 0.733, and skilled worker's wage is 1.47; in g_F these values are respectively 0.784 and 1.568. This, as noted, shows that identical workers may earn (on average) different wages according to their social links.



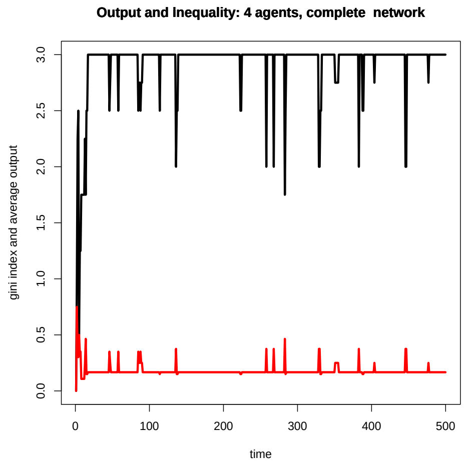


Figure 7: Output and inequality: empty network

Figure 8: Output and inequality: complete network

In both cases the strong negative correlation is clear. In addition, we notice that the (in)stability of output is mirrored in the behavior of the Gini index over time.

The negative correlation depends in general on the parameters. In particular, with a very low breakdown probability with respect to the probabilities of job arrival, we have that the economy is almost always in full employment, corresponding to the maximum per worker output, equal to 3. The levels of wages we chose are rather compressed, and therefore a state of full employment is associated to a low level of inequality. In this case, inequality increases when a worker loses the job, which corresponds to a drop in average output. This explains the negative correlation between output and inequality.

However, the magnitude of this effect is affected by the network structure. In particular, in a comparison of networks g_A, and g_F, we observe that in the complete network the absolute value of the correlation is lower. This is due to the fact that workers in this network are very seldom unemployed, and therefore the system spends relatively long spells of time in states in which output and inequality do not change (see Figure 8), and therefore correlation is absent.

The sign of the correlation may change with different parameters. For example we show in Table 3 that, with a relatively high breakdown probability ($b = 0.5$), the correlation becomes positive in an empty network, and returns to be negative with a path-connected network and a complete network.

g	**Output**	**Inequality**	**Correlation**
g_A	0.843	0.517	0.142
g_E	0.959	0.486	-0.009
g_F	0.961	0.486	-0.015

Table 3: Output and inequality: $b = 0.5$

With a higher probability of losing the jobs, workers are more often unemployed. When all workers are unemployed, inequality is clearly absent. In this case inequality increases when some worker finds a job, and therefore output and inequality move in the same direction. With a positive number of links, the network may counteract the probability of being unemployed, and in practice makes this situation more similar to the case with low b. Once again social links strongly affect correlation between output and inequality even changing its sign. This confirms that social networks play a relevant role in explaining the behavior of such a correlation.

At any rate, we remark that the positive relation between the number of links and output, and the negative relation between the number of links and inequality is robust to the change in b, in particular when we compare g_A with g_E and g_F. [18]

5.2 The role of the network geometry: average path length and the "small world" property

In order to explore the role of the network geometry on output and inequality, we also consider two networks with the same number and type of agents, and the same number of links, reproducing an example of Calvó-Armengol and Jackson (2004) (see Figure 9).

[18] The relation may well be nonmonotonic, as inequality in g_E and g_F is the same. We have also tried with a very high level of production and wage ($y_{22} = 20$ and $y_{22} = 100$) for the match 22, in order to increase wage differentials, without obtaining significant changes in the results. Increasing the number of links still increases output and reduces inequality, the correlation remains strongly negative. With high $y_{22} = 20$ and $b = 0.5$, we obtain a positive correlation with an empty network. With a complete network the correlation is still positive, but lower in absolute value. This confirms that increasing the number of links reduces the correlation, but in the case of high y_{22} and b, correlation is not sufficiently reduced to become negative.

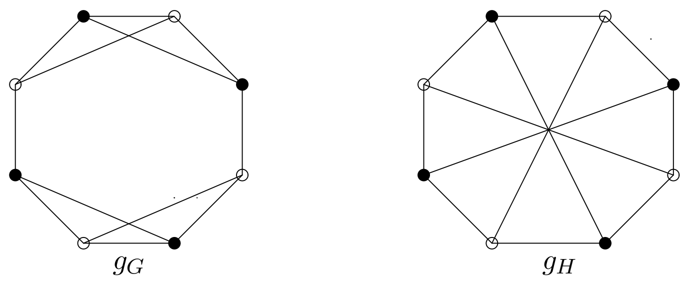


Figure 9: Networks g_G and g_H

In g_G all agents have three links (the two neighbors and a neighbor of one of her/his neighbors) while in g_H they still have three links but one of them is further away. This two networks are characterized by different values of their *average path length*.

In the terminology of the theory of networks (e.g. Albert and Barabasi (2002)), the average path length is the average minimum number of steps to connect any pair of nodes (workers, in our case). In particular, in g_G the average path length is 1.786, while in network g_H is 1.571. Running simulations[19] for these two different networks, we obtain that inequality is approximately identical (0.178), while average output is slightly higher in g_H than in g_G: 2.943 *vs* 2.940.

Network g_H is a simple way to introduce a typical characteristic of real social network, which are referred to as having the "small world" property. The small world property in simple terms refers to the fact that despite the network's size (often large for real world networks), it is possible to find a relatively short path between any two nodes.[20]

In our example, the intuition behind might be synthesized with the fact that "long-range" links facilitate the circulation of information. In particular, for an agent having a link with another worker on the "other side" of the network permits to benefit from the presence of the neighbors of the latter. This because they pass information to the connected worker, increasing the probability that s/he obtains an higher wage and, as a consequence, the probability that s/he passes more job offers to the former connected agent. Of course, the presence of other distant agents could not be exploited (if not marginally) with no link to an agent placed among them.

[19]Parameters are the same of Section 5.1.

[20]More exactly, other than by a short average path length, small world networks are also characterized by an high *clustering coefficient* (see Watts 1999), meaning that agents create dense subgroups highly interconnected (in other words friends of an agent in turn know each other).

5.3 The role of the network geometry: network composition and exclusion

In this section we consider the following issue. Given a structure of the population and a given number of links, which network composition is associated to maximum output and minimum inequality? Are there tradeoffs? Furthermore, which effects on output and inequality does the exclusion of some worker from the network produce?

We consider some possible configurations of a network with eight agents (four skilled and four unskilled, all initially unemployed), with six links. In general, we are considering the plausible situation in which not all possible links exist, given for example the cost of forming the network (see Calvó-Armengol (2004) for an explicit analysis of endogenous network formation with costly links). In Figure 10 we represent various cases in which some agents are excluded from the network.

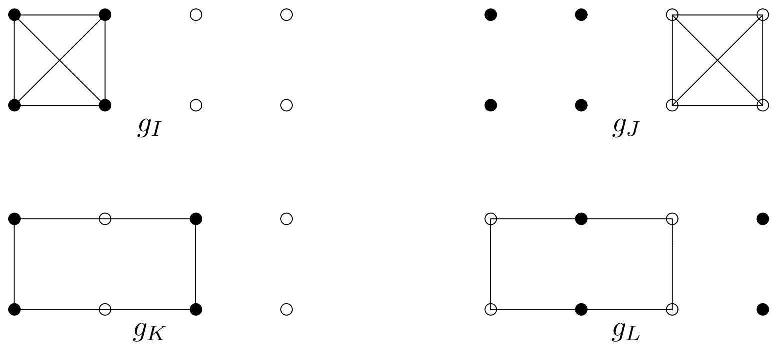


Figure 10: Networks g_I - g_L: exclusion of some agents

In Figure 11, instead, we represent a case with no exclusion, that is in which each agent has at least one link.

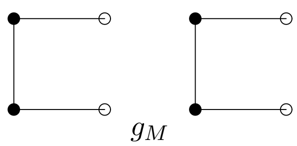


Figure 11: Network g_M: no exclusion

Table 4 summarizes the results of simulations, with the usual parameters.

g	**Output**	**Inequality**	**Correlation**
g_I	2.876	0.201	-0.868
g_J	2.813	0.194	-0.816
g_K	2.890	0.193	-0.851
g_L	2.862	0.190	-0.830
g_M	2.902	0.188	-0.840

Table 4: Output and inequality

In network g_I all unskilled workers are excluded while all skilled workers are connected with each other. This represents a situation with two social groups: the first enjoys a high level of social interaction, while the second is formed by isolated individuals. This situation, associated with the fact that highly connected workers are also the most productive, determines a high level of output but also a high level of inequality. Network g_J represents the polar case: in this situation output and inequality are lower than in network g_I.

In networks g_K and g_L exclusion is partially removed. These networks represent cases in which the population is more mixed, in the sense that agents of different kind share social links. In particular, in g_K two unskilled while in g_L two skilled workers are excluded. With respect to g_I, the inclusion of two unskilled agents in the network, associated with the consequent reduction of the density of links among skilled workers, produces an increase in output and a marked decrease in inequality (the average Gini index drops from 0.201 to 0.193). In a comparison with g_L, instead, output in g_K is higher and inequality is practically the same. Network g_L shows further decrease in inequality, since all unskilled workers have social links and the excluded worker are skilled. However, as expected, this reduces average output.

These results confirm once again that the composition and the geometry of the network play a relevant role in explaining aggregate results, and also that workers with identical observable characteristics have different wage profiles over time according to their social links (for example, in g_I the average wages of skilled and unskilled workers are, respectively, 1.568 and 0.733, while in g_M these values are 1.556 and 0.767).

A particular remark deserves network g_M in which no worker is excluded. The configuration of this network represents the minimal admissible structure with only six links and no worker excluded. The result is particularly interesting: output is the highest and inequality is the lowest. Such a result confirms that social integration can be beneficial in terms of efficiency and equality, given that there is no trade-off in moving from a segregated society (like the one depicted by g_I and g_J) to a more integrated one.[21]

[21] A network similar to g_M in which the pairs of unskilled agents are connected produces

This result is dependent on the assumption on the productivity of the different matches. In particular, it holds if the productivity of skilled workers in good jobs is not very high. In fact, when moving from g_I to g_M, skilled workers are penalized while unskilled workers take advantage from the rewiring of links. The case represented in Table 4 is one in which the second effect dominates the first. Clearly, if the productivity of skilled workers in good jobs is sufficiently high, the result is reversed.

g	**Output**	**Inequality**	**Correlation**
g_I	10.619	0.429	-0.932
g_M	10.563	0.428	-0.959

Table 5: Output and inequality: $y_{22} = 20$

In Table 5 we present the results when $y_{22} = 20$ for a comparison between g_I and g_M. We see that in g_M output decreases and inequality is basically constant.[22] In this example, therefore, to increase the efficiency of the system in terms of production, all advantages of exchanging information on jobs should be reserved to skilled workers. Notice that the negative correlation is particularly high in absolute terms. This result depends on the fact that, when some skilled worker loses the job, output decreases considerably (given the hypothesis on their productivity in good jobs), and inequality increases remarkably as the number of "particularly rich" agents is reduced.[23] In the next section we discuss in more detail this and previous results in order to derive some policy implications.

6 Discussion and policy implications

Our results indicate that network effects are relevant in the labor market since they strongly influence employment perspectives, output and wage inequality as well as correlation between them. In our framework, the most striking indication is that the number of social links produce positive effects both on aggregate production and wage dispersion. In particular, when the number of links increases for a given population of workers and given

a similar result.

[22] In simulations with $y_{22} = 100$, we find that in g_M output and inequality are higher. The result on inequality appears to be dependent on the chosen inequality index, as the distribution improves in the lower percentile and worsens in the highest, but the latter effect dominates. An examination of the dependence of our results on the chosen inequality index is left for future research.

[23] The absolute value of the correlation in g_M increases for reasons outlined above. In g_I the system spends longer spells of time in a "near full employment" state for skilled workers. This produces more stability of output and of inequality for them, hence reducing the correlation in the aggregate.

parameters, indicating that members of a society are more interconnected, output generally increases and inequality decreases. For a given number of links, output may also increase when the average distance among workers is reduced.

This result is not so widespread in the previous literature which, in the presence of social network effects, has often pointed out the existence of a general trade-off between production performance and degree of inequality among workers (e.g. Montgomery (1991)). The reason is due to the fact that in such a literature social networks are primarily a tool of conveying information about workers type or productivity to firms. With such a channel of information, firms become more able to discriminate in hiring and paying among workers and this increases their productivity but also wage dispersion. In our framework, instead, as in Calvó-Armengol and Jackson (2003), social ties permit to transmit information about job vacancies to workers and this produce clear benefits: output increases, since there is an higher probability that workers are employed and are effectively producing, and wage inequality may decrease, since employment perspectives improve for all workers.

The most obvious lesson that derive from such a result is that social ties, or more in general each channel which fosters the transmission of information about job opportunities among workers, should be expanded. Clearly, this depends, at least partially, on our assumptions. For example, firms are totally passive entities in our framework. A natural alternative assumption is that firms "prefer" to allocate good jobs to skilled workers, and therefore are more willing to dismiss unskilled workers in good jobs. The simplest way to consider this aspect would consist in assuming different values of b for different worker-job match. This could possibly cause more inequality, as unskilled workers would be at disadvantage with respect to skilled ones. This, and other extensions, are left for future work.

Another relevant result is represented by the effects produced by the network composition. In other terms, given a population and a fixed number of social ties, which network composition produces better welfare results? Also in this case our results have provided some indications. Networks with links among heterogeneous agents, that is networks which include different type of workers, can be better given some technological requirements. Namely, when the productivity of skilled workers in good jobs is sufficiently low with respect to the productivity in other matches.[24] It can then be posssible that, given a case in which the most productive agents derive the maximum benefit from the social network, allowing some of the less productive agents to have some links, more than compensate the loss of output due to the

[24] Our examples are sufficient to highlight this result. A more detailed analysis of the relationship between productivities in various matches and the dynamics of output and inequality is left for further research.

reduction in the number of links among the most productive agents.

The policy implications may range from residential policies to the organization of the schooling system. Social networks depend heavily on the interaction among individuals and, obviously, neighborhoods and schools are important determinants of the degree of social interaction. In this case, the indication is that more mixed neighborhoods and schools which prevent the exclusion of members of some social group, unskilled in our case but in general any ethnic, religious, or cultural group, can be beneficial to society in terms of higher production and lower inequality.

7 Conclusion

We have proposed a simple model of transmission of information on jobs in a labor market. Workers who share a social relation may exchange information. We have shown that an increase in the number of links is generally associated with an increase in the average level of production and might it be with a decrease in inequality.

In addition, we have studied the effects on output and inequality of the geometry of the network and of its composition. We have shown that, for a given number of links, a network without social exclusion and with a more mixed population sharing social connections can produce a higher level of output and be less unequal, if the difference in productivity between the most productive matches and the others is sufficiently low.

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