# Human Capital, Product Market Power and Economic Growth

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## 1. Introduction

Despite the fact that an important research line of the *Research and Development (R&D)-based* growth literature has already investigated whether the presence of imperfect competition in the product market may be growth-enhancing or not,[1] such an analysis has not yet been conducted within an integrated economic growth model where agents (firms and individuals) may decide to invest respectively in innovation and education activities and the growth engine is the investment in human capital.

This paper aims at combining in the simplest possible way the basic Lucas (1988) model of human capital accumulation with (a version of) the Grossman and Helpman model of endogenous technical change without knowledge spillovers (1991, Ch. 3, pp. 43/57) in order to fill this significant gap in the literature. The reason why we focus on the version of the Grossman and Helpman's model without knowledge spillovers is that we are interested in studying the link between product market competition and economic growth within an economy where the lever to economic development is the investment in formal education, and not the R&D activity.

Apart from introducing explicitly human capital accumulation *à la Lucas*, the structure of our model economy is similar to that of the basic Grossman and Helpman's approach (1991, Ch. 3). In more detail, we assume that there exist three vertically integrated sectors. A competitive final output sector produces a homogeneous consumers good. Depending on the value of a crucial parameter (the share of total income being devoted to the purchase of the available capital good varieties), the final output sector technology may employ (with constant returns to scale) either solely human capital, or solely the existing varieties of intermediates or both as inputs. The intermediate goods sector consists of monopolistically competitive firms, each producing a differentiated variety. We assume that the production of (whatever variety of) intermediate goods requires only human capital. Finally, the research activity produces designs (or blueprints) for new intermediate input varieties by employing only human capital, as well. When a new blueprint is discovered in the competitive R&D sector, an intermediate-good producer acquires the perpetual patent over it. This allows the intermediate firm to manufacture the new variety and

[1] See, among others, Aghion and Howitt (1996, 1998a,b), Aghion, Dewatripont and Rey (1997, 1999), Aghion, Harris and Vickers (1997), Aghion, Harris, Howitt and Vickers (2001), Smulders and van de Klundert (1995), van de Klundert and Smulders (1997), Bucci (2003a). See also Bucci (2003c) for a survey.

practice monopoly pricing forever. Population is stationary and a representative household invests portions of its fixed-time endowment to acquire formal education. Hence, in our model human capital can be used in every sector in order to produce, respectively, a homogeneous final output, capital goods, infinitely-lived patents and new human capital.

Our main conclusions are the following. First of all, we find that there always exist (except when the technology for the production of the homogeneous consumers good is linear in human capital) a positive relationship between product market power and aggregate productivity growth. Secondly, we get that both the *type of technology* being used in the final output sector (Cobb Douglas versus CES) and the way the (growing) human capital is distributed across the different activities (what we term by *inter-sectoral competition for skills*) do affect the relationship between aggregate productivity growth and monopoly power. Lastly, we also show that the *type of technology* being used in the final output sector and the *inter-sectoral competition for human capital* also influence the level of the steady state growth rate.

This paper is especially related to two existing works. Bucci (2003b) also develops an endogenous growth model that integrates purposive R&D activity with human capital accumulation and where the engine of growth is represented by the investment in schooling. The present paper represents a generalisation of Bucci (2003b). The generalisation we propose here consists in writing the production technology in use in the downstream sector in such a way to disentangle the (equilibrium) monopolistic mark-up set in the intermediate sector and the degree of returns to specialization.[2] Due to this generalization, in the present paper we have the possibility of studying in detail, and within the same framework, the relationship between imperfect competition in the product market and economic growth as emerging from two different classes of endogenous growth models: a) the Rebelo's model (1991) with human (instead of physical) capital accumulation (or *"AH model"*), and b) the Grossman and Helpman's model (1991) of endogenous technological change without knowledge spillovers *and* human capital investment (or *"Lucas - Grossman and Helpman's model"*). In other words, the model we present in this paper enables us to analyse the potential implications (as for the equilibrium relationship between monopoly power and economic growth) of the seminal Rebelo's (1991) and

[2] This point is made clear by Benassy (1998, p.63), according to whom the degree of returns to specialisation *"...measures the degree to which society benefits from specialising production between a larger number of intermediates n"*.

Grossman and Helpman's (1991) models when a positive supply of skills is explicitly introduced and to compare such implications with those coming from Bucci (2003b).

The other paper that comes closer to ours is Bucci (2003a). This last paper examines what happens to the market power-growth nexus within a model where there is no human capital accumulation (human capital is in fixed supply) and the engine of growth is represented by the externality in the R&D activity. Unlike Bucci (2003a), in the present article we take an importantly different view, by considering an economy where the lever to economic growth is represented by a deliberate choice of investing in formal education by utility-maximizing agents.

The rest of the paper is organised as follows. Section 2 introduces the basic model. In Section 3 we study the general equilibrium of it and examine its steady-state properties. In Section 4 we compute the equilibrium output growth rate of the economy and solve for the inter-sectoral distribution of human capital. Section 5 presents the results concerning the steady-state predictions of the model about the relationship between the type of production function employed in the downstream sector, the sectoral distribution of human capital, product market power and economic growth in some special cases. Finally, Section 6 concludes.

## 2. The Model Economy

In this economy three vertically integrated sectors produce respectively a homogeneous consumers good, intermediate inputs (capital goods) and ideas. In order to produce the undifferentiated consumers good, an aggregate production function combines with constant returns to scale human capital and intermediate inputs. These are available, at time $t$, in $n_t$ different varieties and are produced by employing only human capital. In the research sector, firms also use human capital to engage in innovation activity. Innovation consists in discovering new designs (or blueprints) for firms operating in the intermediate sector. The number of designs existing at a certain point in time coincides with the number of intermediate input varieties and represents the actual stock of *non-rival* knowledge capital available in the economy. Finally, unlike the traditional *R&D-based growth models*, we assume that the supply of human capital may grow over time. In this respect, and following the path-breaking papers by Uzawa (1965) and Lucas (1988), we postulate the existence of a representative household that devotes part of its own fixed-time endowment to educational activities. Thus, in this economy human capital can be employed to produce consumer goods, intermediate inputs, ideas and new human capital. A complete description of each of these sectors follows.

**The Final Output Sector.**

In this sector atomistic producers engage in perfect competition. The technology to produce final goods (*Y*) is given by:

$$(1) \qquad Y_t = FH_{Yt}^{1-\lambda}\left[\int_0^{n_t}(x_{jt})^{\alpha}\,dj\right]^{\frac{\lambda}{\alpha}}, \qquad F>0, \quad 0 \le \lambda \le 1, \quad 0 \le \frac{\lambda}{1+\lambda} < \alpha < 1.$$

As in Bucci (2003a), we have written the production technology in use in the downstream sector in such a way to disentangle the (equilibrium) monopolistic mark-up set in the intermediate sector and the degree of returns from specialization.[3] Another reason why we employ the production function of equation (1) is that this technology allows to encompass as particular cases (and depending on the value of $\lambda$) two recent models of endogenous growth[4] (one of which is not *R&D-based*) that in their original version do not include human capital accumulation. With respect to these models in this paper we are interested to study their potential implications (as for the monopoly power-growth relationship) when a positive supply of skills is explicitly considered. As already mentioned, unlike Bucci (2003a), we take here a different view by looking at an economy where the lever to economic growth is human capital accumulation (and not the R&D externality).

According to equation (1), output at time *t* ($Y_t$) is obtained by combining, through a constant returns to scale technology, human capital ($H_{Yt}$) and *n* different varieties of intermediate inputs, each of which is employed in the quantity $x_j$. $\alpha$, $\lambda$ and *F* are technological parameters. The

[3] Indeed, in a moment we will show that (under additional assumptions) the mark-up charged over the marginal cost by the monopolistic producers of intermediate inputs is $1/\alpha$. At the same time, from equation (1), it is possible to see that in a symmetric equilibrium (in which the total production of intermediates, X, is spread evenly between the *n* brands) the degree of returns to specialization (the exponent of *n*) is equal to $\lambda(1/\alpha - 1)$. This is clearly different from the monopoly power measure $(1/\alpha)$ and, more importantly, depends not only on $\alpha$ but also on $\lambda$. It is in this specific sense that the model we present here represents an extension of Bucci (2003b).

[4] Namely the Rebelo's (1991) and Grossman and Helpman's (1991, Ch. 3, pp. 43/57) models.

latter (total factor productivity) is strictly positive, whereas $\lambda$ is (not strictly) between zero and one. In a moment we shall see that the restriction on $\alpha$ ensures that in a symmetric equilibrium the instantaneous profit accruing to a generic intermediate producer at a given point in time is inversely related to the number of varieties existing at that date.

Since the industry is competitive, in equilibrium each variety of intermediates receives its own marginal product (in terms of the numeraire good, the final output):

$$(2) \qquad p_{jt} = F\lambda H_{Yt}^{1-\lambda}\left[\int_0^{n_t}(x_{jt})^{\alpha}\,dj\right]^{\frac{\lambda}{\alpha}-1}(x_{jt})^{\alpha-1}.$$

In equation (2) $p_{jt}$ is the inverse demand function faced, at time *t*, by the *j-th* intermediate producer. As it is common in the first generation *innovation-based* growth literature, without any strategic interaction between intermediate input producers (which we assume henceforth) the price-demand elasticity faced by each intermediate firm coincides with the elasticity of substitution between two generic varieties of capital goods and is equal to $1/(1-\alpha)$.

**The Intermediate Goods Sector.**

In the intermediate sector, capital good producers engage in *monopolistic competition*. Each firm produces one (and only one) horizontally differentiated intermediate good and must purchase a patented design before producing its own specialised durable. Following Bucci (2003b), we continue to assume that each local intermediate monopolist has access to the same technology:

$$(3) \qquad x_{jt} = B \cdot h_{jt}, \qquad \forall j \in (0, n_t), \qquad B>0.$$

This production function is characterised by constant returns to scale in the only input employed (human capital) and, according to it, one unit of skills is able to produce (at each time) the same constant quantity of whatever variety. *B* measures the productivity of human capital employed in this sector. The *j-th* intermediate firm maximises (with respect to $x_{jt}$) its own instantaneous profit function under the (inverse) demand constraint (equation 2), and taking as given the human capital input wage rate. Under the assumption that in the intermediate sector there exists no strategic interaction among firms,[5] the resolution of this maximisation program gives the optimal price set by the generic *j-th* intermediate producer for one unit of its own output:

(4) $$p_{jt} = \frac{1}{\alpha}\frac{w_{jt}}{B}.$$

From equations (4) and (2), the wage rate accruing at time $t$ to one unit of human capital employed in the capital goods sector ( $w_{jt}$ ) is equal to:

(4') $$w_{jt} = FB\alpha\lambda H_{Yt}^{1-\lambda}\left[\int_0^{n_t}(x_{jt})^{\alpha}\,dj\right]^{\frac{\lambda}{\alpha}-1}(x_{jt})^{\alpha-1}.$$

In a symmetric equilibrium (where $x_{jt} = x_t$, $\forall j \in (0, n_t)$ ), each local monopolist faces the same wage rate ( $w_{jt} = w_t$, $\forall j \in (0, n_t)$ ) and equation (4) can be recast as:

(4'') $$p_{jt} = \frac{1}{\alpha}\frac{w_t}{B} = p_t, \qquad \forall j \in (0, n_t).$$

The hypothesis of symmetry is suggested by the way each variety of intermediates enters the final output technology and by the fact that all the capital good producers use the same production function (equation 3). Hence, when all intermediate firms are identical, they produce the same quantity ( $x_t$ ), face the same wage rate accruing to intermediate human capital ( $w_{jt}$ ) and fix the same price for one unit of their own output. This price is equal to a constant *mark-up* ( $1/\alpha$ ) over the marginal cost ( $w_{jt}/B$ ). In equilibrium the wage rate accruing to one unit of human capital employed in the intermediate sector ( $w_{jt}$ ) will be the same (and equal to $w_t$ ) for all the sectors where this factor input is employed. This is due to the hypothesis that human capital is homogeneous in this model economy and is perfectly mobile across sectors.

[5] Namely, that $\frac{\partial}{\partial x_{jt}}\left[\int_0^{n_t}(x_{jt})^{\alpha}\,dj\right] = 0$.

Defining by $H_{jt} \equiv \int_0^{n_t} h_{jt} dj$ the total amount of human capital employed in the intermediate sector at time *t*, and under the assumption of symmetry among capital goods producers ($x_{jt} = x_t$, $\forall j \in (0, n_t)$), from equation (3) we obtain:

(5) $$x_{jt} = \frac{B \cdot H_{jt}}{n_t} = x_t, \qquad \forall j \in (0, n_t).$$

Finally, the instantaneous profit function of a generic *j-th* intermediate firm will be:

(6) $$\pi_{jt} = F\lambda(1-\alpha) \cdot H_{Yt}^{1-\lambda} \cdot (n_t)^{\frac{\lambda-\alpha}{\alpha}} \left( \frac{B \cdot H_{jt}}{n_t} \right)^{\lambda} = \pi_t, \qquad \forall j \in (0, n_t).$$

Since we are dealing with a *monopolistic competition market*, $\pi$ will be decreasing in *n* (the number of intermediate firms existing at time *t*) if and only if $\alpha > \lambda / 1 + \lambda$. This explains the restriction on $\alpha$ we have explicitly introduced in equation (1).

Equation (6) says that, just as *x* and *p*, so too the instantaneous profit is equal for each variety of intermediates in a symmetric equilibrium.

**Research and Development Activity.**

There are many competitive research firms undertaking R&D. These firms produce designs indexed by 0 through an upper bound $n \geq 0$ that measures the total stock of society's knowledge. Designs are patented and partially excludable, but non-rival and indispensable for capital goods production. With access to the available stock of knowledge $n$, research firms use human capital to develop new blueprints. The production of new designs is governed by:

(7) $$\dot{n}_t = C \cdot H_{nt}, \qquad C>0,$$

where $n_t$ denotes the number of capital goods varieties existing at time *t*, $H_n$ is the total amount of human capital employed in the sector and *C* is the productivity of the research human capital input. The production function of new ideas in equation (7) coincides with the one employed by Grossman and Helpman (1991) in their Chapter 3 growth model *without knowledge spillovers* (pp.43-57). In that model such a specification of the R&D process implies the cessation of growth in the long run. In our model, instead, this can not happen since in our economy the engine of growth is human capital accumulation. In this last sense the model we present here shares the same conclusions of many other works with purposive R&D activity and skill accumulation.[6]

As the research sector is competitive, imposing the zero profit condition amounts to setting:

(8) $$\frac{1}{C} w_{nt} = V_{nt} ,$$

with:

(9) $$V_{nt} = \int_t^{\infty} \exp\left[ - \int_t^{\tau} r(s) ds \right] \pi_{j\tau} d\tau , \qquad \tau > t .$$

In equations (8) and (9), $w_n$ represents the wage rate accruing to one unit of human capital devoted to research; the term $\exp\left[ - \int_t^{\tau} r(s) ds \right]$ is a present value factor which converts a unit of profit at time $\tau$ into an equivalent unit of profit at time *t*; *r* is the real rate of return on the consumers' asset holdings; $\pi_j$ is the profit accruing to the *j-th* intermediate producer (once the *j-th* infinitely-lived patent has been acquired) and $V_n$ is the market value of one unit of research output (the generic *j-th* idea allowing to produce the *j-th* variety of capital goods). Notice that $V_n$ is equal to the discounted present value of the profit flow a local monopolist can potentially earn from *t* to infinity and coincides with the market value of the *j-th* intermediate firm (in this economy there is a one to one relationship between number of patents and number of capital goods producers).

**The Household Sector.**

Total output produced in this economy (Y) can be only consumed. Population is stationary and the available human capital is fully employed. For the sake of simplicity, we normalize the population size to one and postulate the existence of an infinitely-lived representative consumer with perfect foresight. This consumer owns, in the form of assets (*a*), all the firms operating in the economy and is endowed with one unit of time that he/she allocates (in the fraction *u*) to productive activities (research, capital goods and consumer goods manufacture), and (in the

[6] Notably Arnold (1998) and Blackburn, Hung and Pozzolo (2000).

fraction *1-u*) to non-productive activities (education). The representative consumer maximises under constraint the discounted value of his/her lifetime utility:[7]

$$(10) \qquad \underset{\{c_t, u_t, a_t, h_t\}_{t=0}^{\infty}}{Max} U_0 \equiv \int_0^{\infty} e^{-\rho t} \log(c_t) dt \ , \qquad \rho > 0$$

*s.t.:*

$$(11) \qquad \dot{a}_t = r_t a_t + w_t u_t h_t - c_t$$

$$(12) \qquad \dot{h}_t = \delta(1-u_t)h_t , \qquad \delta > 0$$

$a_0$, $h_0$ given.

The control variables of this problem are $c_t$ and $u_t$, and $a_t$ and $h_t$ are the two state variables. Equation (10) is the intertemporal utility function; equation (11) is the budget constraint and equation (12) represents the human capital supply function.[8] The symbols used have the following meaning: $\rho$ is the subjective discount rate; $c$ denotes consumption of the homogeneous final good; $w$ is the wage rate accruing to one unit of human capital[9] and $\delta$ is a parameter reflecting the productivity of the education technology. With $\mu_{1t}$ and $\mu_{2t}$ denoting respectively the shadow price of the consumer's asset holdings ($a$) and human capital stock ($h$), the first order conditions read as:

$$(13) \qquad \frac{e^{-\rho t}}{c_t} = \mu_{1t} \qquad\qquad (14) \qquad \mu_{1t} = \mu_{2t} \frac{\delta}{w_t}$$

$$(15) \qquad \mu_{1t} r_t = -\dot{\mu}_{1t} \qquad\qquad (16) \qquad \mu_{1t} w_t u_t + \mu_{2t} \delta(1-u_t) = -\dot{\mu}_{2t} .$$

Equation (13) gives the discounted marginal utility of consumption, which satisfies the dynamic optimality condition in equation (15). Equation (14) is the static optimality condition for the allocation of time, equating the marginal benefit and the marginal cost of an additional unit of

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[7] Following Grossman and Helpman (1991) we assume that the instantaneous utility function of the representative agent is logarithmic. Using a more general isoelastic function does not alter the main results of this paper.

[8] We assume no depreciation for human capital. This hypothesis is completely harmless in the present context and serves the scope of simplifying the analysis.

[9] The equilibrium wage rate accruing to human capital is unique since this factor input is perfectly mobile across sectors.

skills devoted to working. The marginal cost involves the cost associated with future reductions in human capital, as expressed by the other dynamic optimality condition in equation (16). Conditions (13) through (16) must satisfy the constraints (11) and (12), together with the two transversality conditions:

$$\lim_{t\to\infty} \mu_{1t} a_t = 0 ; \qquad \lim_{t\to\infty} \mu_{2t} h_t = 0 .$$

This closes the description of our model economy.

## 3. General Equilibrium Analysis and the Steady State of the Model.

In this section we solve for the general equilibrium of the model and characterise its steady state properties under the symmetry hypothesis $(x_{jt} = B \cdot H_{jt} / n_t = x_t ,\ \forall j \in (0, n_t))$. At this aim, after defining by $u*$ the optimal fraction of skills devoted by the representative consumer to production activities,[10] the general equilibrium distribution of human capital between research, capital and consumer goods production can be obtained through solving simultaneously the following equations:

(17) $\quad H_Y + H_j + H_n = u * H , \qquad \forall t$

(18a) $\quad w_j = w_n$

(18b) $\quad w_j = w_Y .$

Equation (17) is the resource constraint, saying that at any time $t$ the sum of the human capital demands coming from each production activity must be equal to the total stock of productive human capital available at the same time. Equations (18a) and (18b) together state that, due to human capital mobility across sectors, in equilibrium the wage earned by one unit of human capital is to be the same irrespective of the sector where it is actually employed.

Moreover, since the total value of the representative agent's assets ($a$) must equal the total value of firms, the next equation has also to be checked in a symmetric equilibrium:

(19) $\quad a = nV_n$

[10] u* is endogenous in the model and, as such, has to be determined.

where $V_n$ is given by equation (9) above and satisfies the asset pricing condition:

$$\text{(19a)} \quad \dot{V}_n = rV_n - \pi_j,$$

with:

$$\text{(19b)} \quad \pi_j = \frac{\lambda}{1-\lambda}(1-\alpha)\frac{H_Y w_Y}{n}, \qquad \text{and} \qquad \text{(19c)} \quad w_Y = \frac{F(1-\lambda)}{H_Y^{\lambda}} n^{\frac{\lambda}{\alpha}} \cdot \left(\frac{BH_j}{n}\right)^{\lambda}.$$

In the model one new idea allows a new intermediate firm to produce one new variety of capital goods. In other words, there exists a one-to-one relationship between number of ideas, number of capital good producers and number of intermediate input varieties. This explains why in equation (19) the total value of the consumer's assets ($a$) is equal to the number of profit-making intermediate firms ($n$) times the market value ($V_n$) of each of them (equal, in turn, to the market value of the corresponding idea). Equation (19a), instead, suggests that the interest on the value of the *j-th* generic intermediate firm $(rV_n)$ should be equal, in equilibrium, to the sum of two terms:

- the instantaneous monopoly profit ($\pi_j$) coming from the production of the *j-th* capital good;
- the capital gain or loss matured on $V_n$ during the time interval *dt* ($\dot{V}_n$).

In order to characterise the steady-state equilibrium of the model presented so far, we start with a formal definition of it:

**Definition: Steady State Equilibrium.**

A steady state equilibrium is an equilibrium where:

a) *the growth rate of all variables depending on time is constant*;
b) *the ratio (R) of human (H) to knowledge (n) capital is constant*, and
c) $H_Y$, $H_j$, $H_n$ *all grow at the same constant rate as H.*

With this definition of steady state equilibrium in mind we notice that, when $g_H$ (the growth rate of $H$) is constant, then $u$ is constant as well (see equation 12).[11] This means that in the steady state the household will optimally decide to devote a constant fraction of its own fixed-time

[11] Given our assumptions on the size of the representative household and the population growth rate, we can easily conclude that $h \equiv H$ (which implies that we can use $g_H$ instead of $g_h$).

endowment to working ($u$*) and educational ($1$-$u$*) activities. Solving explicitly the representative consumer's problem, it is possible to show that the following results do hold in the steady state equilibrium (**See Notes to the Referees <u>NOT</u> to be published for a complete analytical derivation**):

(20) $$r = \frac{\alpha\delta + \lambda(1-\alpha)(\delta-\rho)}{\alpha};$$

(21) $$g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = \delta - \rho;$$

(22) $$g_c = g_a = (\delta-\rho)\left[\frac{\alpha + \lambda(1-\alpha)}{\alpha}\right];$$

(23) $$\frac{H_j}{n} = \frac{\alpha\delta}{C(1-\alpha)};$$

(24) $$\frac{H_Y}{n} = \frac{(1-\lambda)\delta}{\lambda C(1-\alpha)};$$

(25) $$u^* = \frac{\rho}{\delta}.$$

According to result (20), the real interest rate ($r$) is constant. Equation (21) states that in the steady state equilibrium the number of new ideas ($n$), the consumer's total human capital stock ($H$) and the human capital stocks devoted respectively to the final output production ($H_Y$), to the intermediate sector ($H_j$) and to research ($H_n$) all grow at the same constant rate, given by the difference between the schooling technology productivity parameter ($\delta$) and the subjective discount rate ($\rho$). Equation (22) gives the equilibrium growth rate of consumption and the consumer's asset holdings. Equations (23) and (24), instead, give respectively the equilibrium values of the constant $H_j / n$ and $H_Y / n$ ratios, whereas equation (25) represents the optimal and constant fraction of the representative agent's time-endowment that he/she will decide to devote to working activities ($u$*). For the growth rate of the variables in equations (21) and (22) to be positive and bounded, $\delta$ should be strictly greater than $\rho$ and bounded. The condition $\delta > \rho$ also assures that $0 < u^* < 1$.

## 4. Endogenous Growth and the Shares of Human Capital devoted to the different activities.

To compute the output growth rate of this economy in a symmetric, steady state equilibrium we first rewrite equation (1) as follows:

$$Y_t = FH_{Yt}^{1-\lambda} n_t^{\frac{\lambda}{\alpha}} \left( \frac{B \cdot H_{jt}}{n_t} \right)^{\lambda} = \Psi H_{Yt}^{1-\lambda} n_t^{\frac{\lambda}{\alpha}}, \qquad \Psi \equiv F\left( \frac{B \cdot H_{jt}}{n_t} \right)^{\lambda}.$$

Then, taking logs of both sides of this expression, totally differentiating with respect to time and recalling that in the steady state equilibrium $g_{H_Y} = g_n = g_H = \delta - \rho$ (see equation 21 above), we obtain:

$$\text{(1a)} \qquad \frac{\dot{Y}_t}{Y_t} \equiv g_Y = g_c = g_a = \left[ \frac{\alpha + \lambda(1-\alpha)}{\alpha} \right] g_H = \left[ 1 + \lambda(\beta - 1) \right] \cdot (\delta - \rho), \qquad \beta \equiv 1/\alpha > 1.$$

Hence, economic growth depends only on $\alpha$ (the inverse of which can be easily interpreted as a measure of the monopoly power enjoyed by each intermediate local monopolist[12]), $\lambda$ (which represents the share of total income being devoted in a symmetric equilibrium to the purchase of the available capital goods varieties[13]) and the accumulation rate of human capital ( $g_H$ ). In this last respect the model supports the main conclusion of that branch of the endogenous growth literature pioneered by Uzawa (1965) and Lucas (1988).

In equation (1a), the term $\lambda(\beta - 1)$ measures the *returns to specialization*. Such returns positively depend not only on $\beta$ (the monopoly power), but also on $\lambda$. The intuition behind this result is as follows: the higher the mark-up rate that can be charged over the marginal cost in the monopolistic sector and the higher the share of national income spent on the intermediate inputs, the higher the return an intermediate producer may obtain from specialising in the production of the marginal variety of capital goods. Moreover, it is also worth pointing out that $\beta$ enters the

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[12] The higher $\alpha$, the higher the elasticity of substitution between two generic intermediate inputs. This means that they become more and more alike when $\alpha$ grows and, as a consequence, the price elasticity of the derived demand curve faced by a local monopolist tends to be infinitely large when $\alpha$ tends to one. Thus, the inverse of $\alpha$ ($1/\alpha$) can be considered as a measure of how uncompetitive the capital goods sector is (see Aghion and Howitt, 1997, p.284).

[13] $\lambda \equiv \frac{\int_0^{n_t} (p_{jt} \cdot x_{jt}) dj}{Y_t}$, with $p_{jt} = p_t$ and $x_{jt} = x_t$, $\forall j \in (0; n_t)$.

equilibrium growth rate when (and only when) $\lambda$ is not equal to zero (i.e. when capital goods are an input in the production of the final good). The reason why it is to be so is clear when one recalls that the only product market where imperfect competition prevails in the model is the intermediate one.

Before computing the shares of human capital devoted to the different economic activities, we first need to determine an expression for the equilibrium human to technological capital ratio ($R \equiv H/n$). At this aim, we use equation (17), with $u^* = \rho/\delta$, $H_j/n = \alpha\delta/(1-\alpha)C$ and $H_Y/n = (1-\lambda)\delta/\lambda C(1-\alpha)$, and obtain:

$$\frac{H_n}{n} = R\frac{\rho}{\delta} - \frac{\alpha\delta}{C(1-\alpha)} - \frac{(1-\lambda)\delta}{(1-\alpha)\lambda C},$$

which, in turn, implies:

$$(26) \qquad g_n = C\frac{H_n}{n} = RC\frac{\rho}{\delta} - \frac{\alpha\delta}{(1-\alpha)} - \frac{(1-\lambda)\delta}{\lambda(1-\alpha)}.$$

Equating this last expression to equation (21) yields:

$$(27) \qquad R \equiv \frac{H_t}{n_t} = \frac{\delta[\delta - \lambda\rho(1-\alpha)]}{\lambda\rho(1-\alpha)C}, \quad \forall t.$$

Given *R*, the shares of human capital devoted to each sector employing this factor input in the decentralised, symmetric, steady state equilibrium can be easily determined as follows:

$$(28) \qquad s_j \equiv \frac{H_j}{H} = \frac{H_j}{n}\cdot\frac{n}{H} = \frac{H_j}{nR} = \frac{\alpha\lambda\rho}{\delta - \lambda\rho(1-\alpha)};$$

$$(29) \qquad s_Y \equiv \frac{H_Y}{H} = \frac{H_Y}{n}\cdot\frac{n}{H} = \frac{H_Y}{nR} = \frac{\rho(1-\lambda)}{\delta - \lambda\rho(1-\alpha)};$$

$$(30) \qquad s_n \equiv \frac{H_n}{H} = \frac{H_n}{n}\cdot\frac{n}{H} = \frac{H_n}{nR} = \frac{\lambda\rho(\delta-\rho)(1-\alpha)}{\delta[\delta - \lambda\rho(1-\alpha)]};$$

$$(31) \qquad s_H \equiv \frac{H_H}{H} = 1 - u^* = \frac{\delta - \rho}{\delta}.$$

Thus, the shares of human capital devoted to each activity depend on the technological ($\lambda$ and $\delta$) and preference ($\rho$) parameters and also on the degree of competition in the capital goods sector ($\alpha$).[14]

## 5. Technology, Sectoral Distribution of Human Capital and the Interplay between Product Market Power and Economic Growth.

All the results stated up to now have been obtained under the assumption that $\delta$ is strictly greater than $\rho$ and bounded. In the present section, while continuing to keep these assumptions, we study how the sectoral shares of human capital and the relationship between product market power and economic growth may change when $\lambda$ is assumed to be respectively equal to zero, one and $\alpha$ (i.e., when we allow the production function in the downstream sector to change).

### Case (a): $\lambda = 0$.

In this case the technologies adopted in each economic sector (in the symmetric, steady state equilibrium) are:

$Y_t = AH_t$, $\quad A \equiv \frac{F\rho}{\delta}$ (for the final goods production);

$x_{jt} = B \cdot h_{jt}$, $\quad \forall j(0, n_t)$ (for the capital goods production);

$\dot{n}_t = C \cdot H_{nt}$ (for research);

$\dot{h}_t = (\delta - \rho) \cdot h_t$ (for human capital supply),

and the model we are dealing with is the Rebelo (1991)-Lucas (1988) one or "*AH*-model". The main variables of the model take on the following values:

$s_j = 0$; $\quad s_Y = \frac{\rho}{\delta}$; $\quad s_n = 0$; $\quad s_H = \frac{\delta - \rho}{\delta}$; $\quad r = \delta$;

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14 See Bucci (2002) for a thorough analysis of the way these four variables (respectively $\lambda$, $\delta$, $\rho$ and $\alpha$) may influence the across-sectors distribution of human capital. It is outside the scope of this work to discuss in detail such comparative statics results.

(32) $\quad g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = g_c = g_a = g_Y = \delta - \rho$.

As is well known, both in Rebelo (1991) and Lucas (1988) technical progress happens through devoting resources to physical (human) capital accumulation rather than a deliberate R&D activity aimed at expanding the set of available (horizontally differentiated) capital goods. In the case under analysis this is reflected in the fact that the intermediate inputs do not enter the final goods production technology and $s_j = s_n = 0$. Thus, all the human capital is distributed between the final output ($s_Y$) and education ($s_H$) sectors. Since capital goods are not productive inputs, market power ($1/\alpha$), which in the model outlined in the previous sections arises from the intermediate sector, does not play any role on the growth rate of output ($g_Y$). As in Lucas (1988), this last coincides with the growth rate of human capital and is equal to the difference between the productivity of the schooling technology ($\delta = r$) and the subjective discount rate ($\rho$).[15] Finally, it is worth noticing that in the steady state equilibrium (when each sector gets a constant fraction of the available stock of human capital) $s_Y$ affects only the level of output ($Y_t = Fs_Y H_t$), whereas its growth rate is solely driven by $s_H$ ($g_Y = \delta \cdot s_H$).

### Case (b): $\lambda = 1$.

When $\lambda = 1$ the technologies employed in each economic sector (in the symmetric, steady state equilibrium) can be recast as:

$$Y_t = F\left[\int_0^{n_t} (x_{jt})^\alpha \, dj\right]^{\frac{1}{\alpha}}, \qquad \text{(for the final goods production);}$$

$$x_{jt} = B \cdot h_{jt}, \qquad \forall j(0, n_t) \qquad \text{(for the capital goods production);}$$

$$\dot{n}_t = C \cdot H_{nt} \qquad \text{(for research);}$$

$$\dot{h}_t = (\delta - \rho) \cdot h_t \qquad \text{(for human capital supply),}$$

[15] See Barro and Sala-I-Martin (1995, p.184, equation 5.29). In our case the elasticity of intertemporal substitution equals one.

and the model we are dealing with is the Lucas (1988)-Grossman and Helpman (1991, Chap. 3, pp. 43/57) one. The main variables of the model now take on the following values:

$s_j = \dfrac{\alpha\rho}{\delta - \rho(1-\alpha)}$; $\qquad s_Y = 0$; $\qquad s_n = \dfrac{\rho(1-\alpha)(\delta-\rho)}{\delta[\delta - \rho(1-\alpha)]}$;

$s_H = \dfrac{\delta-\rho}{\delta}$; $\qquad r = \dfrac{\alpha\delta + (1-\alpha)(\delta-\rho)}{\alpha}$; $\qquad g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = \delta - \rho$;

(33) $\quad g_c = g_a = g_Y = \dfrac{\delta-\rho}{\alpha}$

In this case human capital enters only indirectly (through the capital goods) the final output technology, whereas it continues to be employed in all the remaining sectors ($s_Y = 0$ and $s_j$, $s_n$ and $s_H$ are all positive). As in the previous case, the accumulation rate of human capital is equal in equilibrium to $\delta - \rho$, but now the growth rate of output ($g_Y$) depends positively on the mark-up rate ($1/\alpha$). The reason is that in the present case $g_Y$ is a function not only of $s_H$, but also of $s_n$:

$$g_Y = \frac{(1-\alpha)\rho\delta \cdot s_n}{\rho(1-\alpha) - \delta \cdot s_n} + \delta \cdot s_H ,$$

and it is easy to show that $\dfrac{\partial s_n}{\partial(1/\alpha)} > 0$ and $\dfrac{\partial g_Y}{\partial s_n} > 0$. In other words, in this particular case it is through allocating a higher share of human capital from the intermediate sector ($\partial s_j / \partial(1/\alpha) < 0$) towards the research sector that monopoly power positively affects aggregate economic growth.

### Case (c): $\lambda = \alpha$ .

The last special case we wish to deal with in this section is the case where $\lambda = \alpha$ .[16] Under this assumption the technologies adopted in each economic sector (in the symmetric, steady state equilibrium) become:

$$Y_t = FH_{Yt}^{1-\alpha} \cdot \int_0^{n_t} (x_{jt})^{\alpha}\, dj , \qquad \text{(for the final goods production);}$$

[16] This is the only case considered in Bucci (2003b), where the mark-up rate and the returns to specialization are not disentangled (they both depend exclusively on $\alpha$ in a symmetric, steady state equilibrium).

$x_{jt} = B \cdot h_{jt}$, $\quad \forall j(0, n_t)$ (for the capital goods production);

$\dot{n}_t = C \cdot H_{nt}$ (for research);

$\dot{h}_t = (\delta - \rho) \cdot h_t$ (for human capital supply).

The main variables of the model take on the following values:

$$s_j = \frac{\alpha^2 \rho}{\delta - \alpha\rho(1-\alpha)}; \qquad s_Y = \frac{\rho(1-\alpha)}{\delta - \alpha\rho(1-\alpha)}; \qquad s_n = \frac{\alpha\rho(1-\alpha)(\delta-\rho)}{\delta[\delta - \alpha\rho(1-\alpha)]};$$

$$s_H = \frac{\delta - \rho}{\delta}; \qquad r = \delta(2-\alpha) - \rho(1-\alpha); \qquad g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = \delta - \rho;$$

(34) $\quad g_c = g_a = g_Y = (2-\alpha)(\delta - \rho)$

In the present case human capital is employed in each economic sector. Thus, we can identify this case (unlike the two previous ones) as that in which the inter-sectoral competition for the same input (human capital) is tougher ($s_j, s_Y, s_n$ and $s_H$ are all positive). As before, the accumulation rate of human capital is equal in equilibrium to $\delta - \rho$, but now (unlike case b) the relationship between $s_n$ and $1/\alpha$ is non-monotonic[17] and the growth rate of output ($g_Y$) is a positive and non-linear (concave) function of the mark-up rate ($1/\alpha$).[18]

The main results of the model concerning the relationship between the shape of the production technology in use in the downstream sector, the sectoral distribution of human capital, the degree of product market power and the aggregate economic growth rate can be summarised as follows:

### Result 1:

*Within a generalised growth model of deterministic and horizontal R&D activity where economic growth is sustained by a supply function of skills à la Lucas (1988), as the one described by the steady state equilibrium equations (20) through (31) and (1a), there always exists (except when $\lambda = 0$) a positive relationship between product market power ($1/\alpha$) and aggregate economic growth ($g_Y$).*

[17] See Bucci (2003b, pp. 274-75) for an intuition.

[18] Again, see Bucci (2003b, pp. 277-78) for further details.

**Proof:**

*See equations (1a), (32), (33) and (34).*

The reason why there exists no relationship between market power and growth when $\lambda = 0$ is that in this case there is neither an intermediate sector, nor a research one (accordingly, the output growth rate is completely independent of the mark-up that in the model arises from the capital goods sector).

What this result suggests is the following: as far as the steady state equilibrium relationship between the degree of product market power and aggregate economic growth is concerned, we can replicate one of the most important conclusions obtained in the basic neo-Schumpeterian model[19] simply by using a horizontal product differentiation approach where: 1) human and technological capital may grow at the same constant and positive rate; 2) the engine of growth is human capital accumulation, and 3) there exists no pecuniary externality from purposive R&D activity.

## Result 2:

*Within a generalised growth model of deterministic and horizontal R&D activity where economic growth is sustained by a supply function of skills à la Lucas (1988), as the one described by the steady state equilibrium equations (20) through (31) and (1a), both the type of technology being used in the final output sector and the intensity of the inter-sectoral competition for the (growing) human capital affect the shape of the relationship between aggregate economic growth ( $g_Y$ ) and the monopoly power ( $1/\alpha$ ).*

Indeed, such a relationship is linear in the Lucas-Grossman and Helpman model (case b) - where human capital is not directly employed in the final output sector, whose technology is of the CES type - and concave in case (c), where human capital is used everywhere and the final output technology is (an extension of) Cobb/Douglas. Similar results are obtained in a model where the growth engine is the R&D externality and there is no human capital accumulation (see Results 1 and 2 in Bucci, 2003a).

[19] Namely, Aghion and Howitt (1992).

**Result 3:**

*Within a generalised growth model of deterministic and horizontal R&D activity where economic growth is sustained by a supply function of skills à la Lucas (1988), as the one described by the steady state equilibrium equations (20) through (31) and (1a), both the type of technology being used in the final output sector and the intensity of the inter-sectoral competition for the (growing) human capital affect the steady state equilibrium growth rate. This last is higher whenever the final output technology is CES and does not employ human capital.*

**Proof:**

*From equations (32), (33) and (34) one easily concludes that:* $g_Y$ *(case b)* > $g_Y$ *(case c)* > $g_Y$ *(case a).*

This result parallels Results 3 and 4 of Bucci (2003a). Therefore, even when human capital is allowed to grow over time, the highest possible steady state economic growth rate is obtained within a *Grossman and Helpman-type economy*. On the contrary, the lowest steady state economic growth rate prevails in a *Rebelo/Lucas-type economy*, where the final output technology is linear in the human capital input and all the existing markets (final output and education) are perfectly competitive.

## 6. Concluding Remarks

In this article we presented a generalization of Bucci (2003b). The generalisation we proposed has consisted in writing the production technology in use in the downstream sector in such a way to disentangle the (equilibrium) monopolistic mark-up set in the intermediate sector and the degree of returns to specialization. Depending on the value of a specific parameter (the share of total income being devoted to the purchase of the available capital good varieties), we were able to study the relationship between product market power and economic growth as it emerges from two different classes of endogenous growth models: a) the Rebelo's model (1991) with human (instead of physical) capital accumulation, and b) the Grossman and Helpman's model (1991, Chap.3) of endogenous technological change without knowledge spillovers *and* human capital investment. At the same time, the proposed generalization allowed us to encompass the model of Bucci (2003b) as a special case and to analyse the impact that both the kind of technology in use in the downstream sector and the degree of *inter-sectoral competition for the (growing) human* *capital* have on the market power/economic growth nexus and the level of the steady state equilibrium growth rate in the presence of human capital accumulation, the growth engine. In this last respect, we compared our results with those obtained in Bucci (2003a), where human capital is in fixed supply and economic growth is driven solely by the positive externality from the R&D activity.

Our main findings were threefold. First of all, we found that the presence of more intense product market power within the sector producing capital goods turns out to have always positive growth effects (except when the share of national income spent on the purchase of capital goods is exactly equal to zero). This confirms one of the results found by Bucci (2003b), according to which it is possible to restore the *Schumpeterian growth paradigm* provided that: 1) human capital accumulation (*à la Lucas*) is the engine of growth; 2) there exists no pecuniary externality from purposive R&D activity, and 3) human and technological capital may grow at the same constant and positive rate in the steady state equilibrium. Secondly, we obtained that, though positive, the relationship linking product market power and economic growth may be linear or concave depending on the type of technology employed in the final output sector (CES versus Cobb-Douglas) and the way human capital is distributed across sectors (whether this factor input is employed in every economic activity or not). Finally, we showed that these two elements (the type of technology in use in the final output sector and the intensity of the inter-sectoral competition for human capital) are also able to affect the level of the steady state growth rate. This is higher within a *Grossman/Helpman/Lucas-type* economy where the final output technology is CES and does not employ human capital directly.

Our findings depend on the hypothesis (common to all the first-generation innovation-based growth models) that there exists no strategic interaction among rivals on goods and factor markets. In the future it could be interesting to analyse how, within the framework proposed in this paper, the market power-growth relationship might change when one explicitly allows for the presence of some kind of interaction among firms.

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## NOTES TO THE REFEREES NOT TO BE PUBLISHED.

In these Notes, I derive the set of results (20) through (25) in the main text.

From equation (12), when the rate of human capital accumulation ( $g_h$ ) is constant $u_t$ turns out to be constant as well. This means that in the steady state equilibrium the household will decide to devote a constant fraction of its own time endowment to working (*u*) and educational (*1-u*) activities. Consequently, the optimal *u* (denoted by *u**) will be constant and endogenously determined through the solution to the representative consumer's problem. Consider now this problem (equations 10 through 12 in the main text), whose first order conditions (equations 13 through 16) are reported below for convenience, together with the consumer's constraints and the transversality conditions:

(11) $\dot{a}_t = r_t a_t + w_t u_t h_t - c_t$ (12) $\dot{h}_t = \delta(1-u_t)h_t$, $\delta > 0$

(13) $\dfrac{e^{-\rho t}}{c_t} = \mu_{1t}$, $\rho > 0$ (14) $\mu_{1t} = \mu_{2t}\dfrac{\delta}{w_t}$

(15) $\mu_{1t} r_t = -\dot{\mu}_{1t}$ (16) $\mu_{1t} w_t u_t + \mu_{2t}\delta(1-u_t) = -\dot{\mu}_{2t}$

$\lim_{t\to\infty} \mu_{1t} a_t = 0$ $\lim_{t\to\infty} \mu_{2t} h_t = 0$.

$a_0$, $h_0$ given.

From now on I will omit the index *t* near the time-dependant variables. Combining equations (14) and (16) we get:

(1) $\dfrac{\dot{\mu}_2}{\mu_2} = -\delta$,

whereas, from (15):

(2) $\dfrac{\dot{\mu}_1}{\mu_1} = -r$.

In a symmetric, steady state equilibrium $H_Y$, $H_j$, $H_n$ and *n* all grow at the same constant rate as $H$ (denoted by $g_H$). This in turn implies that:

- *x* (the output produced by each local monopolist, and equal to $BH_j/n$) is constant over time;

- from equations (4'), (19c) and (18b) in the main text, the wage rate accruing to one unit of skilled labour ($w_j = w_Y = w$) grows at a rate equal to $\left[\frac{\lambda(1-\alpha)}{\alpha} g_H\right]$.

Then, using equation (14) in these Notes, we get:

(3) $\frac{\dot{\mu}_1}{\mu_1} = \frac{\dot{\mu}_2}{\mu_2} - \frac{\lambda(1-\alpha)}{\alpha} g_H \quad \Rightarrow$

(3') $r = \delta + \frac{\lambda(1-\alpha)}{\alpha} g_H$.

This means that in the steady state equilibrium (when $g_H$ is constant), the real interest rate (*r*) is constant as well. From equation (6) in the main text it follows that the instantaneous profit accruing to each capital good producer also grows at the rate $\frac{\lambda(1-\alpha)}{\alpha} g_H$. Hence, through simple algebraic manipulations, equation (9) in the main text may be recast as:

(9a) $V_{nt} = F\lambda(1-\alpha)\left(\frac{BH_{jt}}{n_t}\right)^{\lambda} \frac{H_{Yt}^{1-\lambda} \cdot n_t^{\frac{\lambda-\alpha}{\alpha}}}{\delta}$.

According to the equation above, the market value (the discounted flow of future profits) of a generic *j-th* intermediate firm (equal to the market value of the corresponding *j-th* idea) grows in the steady state equilibrium at the rate $\frac{\lambda(1-\alpha)}{\alpha} g_H$. Using equations (8) in the main text and (9a) above, it is possible to conclude that:

(9b) $w_{nt} = FC\lambda(1-\alpha)\left(\frac{BH_{jt}}{n_t}\right)^{\lambda} \frac{H_{Yt}^{1-\lambda} \cdot n_t^{\frac{\lambda-\alpha}{\alpha}}}{\delta}$.

Employing equations (18a) and (4') in the main text and equation (9b) above, we get:

(4) $\frac{H_j}{n} = \frac{\alpha\delta}{C(1-\alpha)}$,

whereas using equations (18b), (4') and (19c) in the main text, the result comes out that:

(5) $\frac{H_Y}{n} = \frac{1-\lambda}{\alpha\lambda} \cdot \frac{H_j}{n} = \frac{(1-\lambda)\delta}{\lambda C(1-\alpha)}$.

Combining equations (13) and (15), we find the usual Euler equation, giving the optimal household's consumption path:

(6) $\frac{\dot{c}}{c} \equiv g_c = r - \rho = \delta - \rho + \frac{\lambda(1-\alpha)}{\alpha} g_H$.

Dividing both sides of equation (11) by *a*, we get:

(7) $\frac{c}{a} = r + wu\frac{h}{a} - g_a$.

We already know that in the steady state equilibrium *r*, *u* and $g_a$ are constant. Hence, for the ratio *c/a* to be constant it should be the case that $wh/a$ is constant. Indeed, *h* grows at the rate $g_H$, $w$ $(= w_Y = w_j = w_n)$ grows at the rate $\frac{\lambda(1-\alpha)}{\alpha} g_H$ and $g_a = \frac{\lambda(1-\alpha)+\alpha}{\alpha} g_H$.[20] Thus, we can conclude that in equilibrium the growth rate of $wh/a$ is equal to zero and the ratio *c/a* is constant. In other words, household's consumption (*c*) and asset holdings (*a*) grow at the same constant rate in the steady state equilibrium. This rate is equal to:

(8) $g_c = g_a = r - \rho = \delta - \rho + \frac{\lambda(1-\alpha)}{\alpha} g_H$.

Finally, to find out the optimal *u** one first equates equation (6) in these Notes with the value of $g_a$ given above,[21] and obtains:

(9) $g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = \delta - \rho$.

Then, plugging equation (9) into (12):[22]

(10) $\frac{\dot{h}}{h} \equiv g_H = \delta(1-u) = \delta - \rho \qquad \Rightarrow \qquad u^* = \frac{\rho}{\delta}$.

For $g_H$ to be strictly positive, $\delta$ should be strictly greater than $\rho$, which in turn implies $0 < u^* < 1$. When $g_H = \delta - \rho > 0$, the real interest rate and the growth rate of consumption and asset holdings become respectively:

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[20] This follows immediately from the fact that $a = nV_n$ (equation 19 in the main text), $g_n = g_H$ (in the steady state equilibrium) and $g_{V_n} = \frac{\lambda(1-\alpha)}{\alpha} g_H$ (see equation 9a in these Notes).

[21] $g_a = \frac{\lambda(1-\alpha)+\alpha}{\alpha} \cdot g_H$.

[22] Notice that in equation (10) we explicitly use the fact that $g_H = g_h$. This is so because in our model the representative household has unit measure and there is no population growth.

(3'') $r = \dfrac{\alpha\delta + \lambda(1-\alpha)(\delta-\rho)}{\alpha} > 0$;

(8') $g_c = g_a = r - \rho = (\delta - \rho)\left[\dfrac{\alpha + \lambda(1-\alpha)}{\alpha}\right]$>0.

Also notice that when $r$ and $g_a$ take on the values written above and $g_H = \delta - \rho$, then the two transversality conditions are trivially checked in the steady state equilibrium since:

$$\lim_{t\to\infty} \mu_{1t} \cdot a_t = \mu_{10} \cdot a_0 \cdot \lim_{t\to\infty} e^{-\rho t} = 0;$$

$$\lim_{t\to\infty} \mu_{2t} \cdot h_t = \mu_{20} \cdot h_0 \cdot \lim_{t\to\infty} e^{-\rho t} = 0.$$