Co-evolution of population and natural resources: a simple Malthusian-Ricardian model

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23rd January, 2004

Preliminary Draft

Abstract

The History of Easter Island Civilization attracted several authors after Brander and Taylor (1998). Most of these studies are simple variations of a standard Ricardian model of growth, in particular we show that these kind of models have the same structure of Pasinetti (1960) except for the fact that they add a renewable natural resource to the traditional representation of Ricardian system. Moreover, through this model we obtain some new interesting behaviour of the dynamics of the system. Indeed, the co-evolution of population and a renewable natural resource generates two different regimes and therefore two different path of development; in the first one, a stable sustainable equilibrium is attained with positive values for both population and resource stock; while in the second regime the renewable resource is completely exhausted although the population survive at a positive level, consistently with the conditions observed in Easter Island by the first European explorers. Furthermore, we analyse which are the possible reasons which could lead the Easter Island Civilization to switch from the sustainable to the unsustainable regime. We have recognised four reasonable causes: technical progress, change on preferences, social and ecological discontinuities.

Keywords: Population; Renewable resources; Technical progress; Resource scarcity

I. Introduction

The existence of natural limits to economic growth has been increasingly investigated in the last decades. The main point of this debate is whether or not the current choices about the path of development have significant effects on future wellbeing. Recent studies underline that the degradation of natural resources had strong blowbacks on the decline of many societies[1] as it is stressed in Brander and Taylor (1998) (hereafter B/T).

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[1] See for instance Huntington (1917), Tainter (1988), Tong (1988), Goldstone (1991), Anderies (1998) that take into account different examples of resource scarcity as the main cause of the decline of past society.

The main objective of this paper is to study the co-evolution between the growth of population and the deterioration of a renewable resource. The history of Easter Island civilization is suitable to investigate this topic; indeed several recent studies support the thesis that the employment of the palm forest as basic natural resource was the crucial element for the development of this civilization (see for instance Bahn and Flenley, 1992 and Van Tilberg, 1994). Palms were useful to make canoes (to catch fishes) but also to transport the big statues (called "Moai"). Unfortunately the regeneration of palm forest is quite slow and the intense exploitation leaded the society to face a grim shortage. As Kirch (1984, p.264) underlines "Easter Island is a story of a society which – temporarily but brilliantly surpassing its limits – crashed devastatingly"[2]. The reason of that decline can be traced back to the "Malthusian Trap"[3] as many authors pointed out (Weiskel, 1989; Ponting, 1991; Keegan, 1993; Brown and Flavin, 1999).

The B/T article originated a long sequence of ecological-economic studies on the Easter Island civilization (Reuveny and Decker, 2000; Dalton and Coats, 2000; Erickson and Gowdy, 2000; Pezzey and Anderies, 2003). Our model is inspired by the B/T article although it performs many variations which make the basic structure of the model approach more markedly the Classical tradition. Indeed, the development of this model has a clear Ricardian derivation; the production structure is the same of Pasinetti (1960) except for the addition of the natural resource in the sector which produces the luxury good. Hence, the model allows us to analyse how a simplified version of a Ricardian system evolves in the presence of a renewable and exhaustible resource.

The main result of the model is that the dynamical system shows two different regimes. The first is sustainable[4] in the long run and it allows the population to have constant positive utility; the second one is not sustainable, the utility at equilibrium is nil but the system allows population to survive. This solution is not obtained by B/T, but is consistent with the fact that (few thousands) survivors were found in Easter Island by the first European explorers.

It is interesting to analyse which conditions make the system to flip from one to the other regime. One important cause is technical progress. In the representation of the Ricardian system given by Pasinetti (1960) the role of technical progress is not considered in the long period analysis. Pasinetti (1960, p.86) explains that Ricardo "… only points out that improvements in the technical conditions postpone in time the effects of the changes of type (ii) [the accumulation process]. Since he thinks that these changes (capital accumulation) are – in order of magnitude – the more relevant ones, he concentrates his analysis on them, with the qualification that the effects he shows might be delayed, thought not modified, by technical progress". Instead, in our model the effect of technical progress is different: we prove that an exogenous improvement of the technology in both sectors is an incentive to a faster degradation of natural resources and it can switch the economy from one regime with positive resource stock in equilibrium to another with zero resource stock. This result stresses that technical progress alone cannot save an economy from big blowbacks.

[2] See also Brander and Taylor (1998, p.122).

[3] From the famous Malthus's (1798) book "An essay on the principle of population".

[4] The most common definition of sustainability is by WCED (1987): "Sustainable Development is development that meets the needs of present without compromising the ability of future generations to meet their own needs". The translation of this concept in economic modelling is often reduced to the formula: the utility of future generations must be at least at the same level than the utility of current generation. During this paper we maintain this simplicistic view of sustainability.

From the other side, the preferences here have the crucial role of determining in which of the two regimes we are, but an exogenous change in preferences can lead the system towards the other regime. Moreover, we have two kinds of discontinuities that should be considered; one kind is connected to social factors, since the main idea is that when the society faces a deep scarcity of palm forest the social relations tend to collapse and conflicts between groups become more plausible[5]. The second kind of discontinuities is given by the ecosystem dynamics; as we know, we cannot suppose that the extinction of trees on the Island would leave the whole natural ecosystem undamaged.

Our theoretical approach, through the comprehension of the most important causes that determine which regime emerges, allows us to study qualitatively different paths of development. Therefore, Ricardian-Malthusian models of growth must be considered a suitable instrument to analyse the co-evolution between growth of population and the dynamics of renewable natural resources.

The rest of the paper is organized as follows. Section II presents the assumptions of the model and their meaning; sections III and IV are dedicated to the development of the model; section V describe the two different regimes; sections VI and VII investigate respectively the role of technical progress and of preferences; then we draw some concluding remarks.

II. Building the model

In the economy of Easter Island, the wood of palm forest is the crucial element allowing the civilization to grow rapidly, and the role of this specific resource should be well stressed in the model. Therefore, to simplify the analysis we decide to aggregate the production sectors of Easter Island economy in two groups. The first considers all the processes which do not use palm forest, that we call the traditional sector because it is formed by those products (plants, animals and kind of food) which were transported by Polynesian Civilization to colonize a new island.

The second sector is based on the palm forest, since Easter Island civilization used the wood of that tree for two main uses; first of all to built canoes and catch fishes (especially porpoise), but also to transport the "Moai" the famous statues strewn all over the island, making them roll over the palm trunks.

Hence we consider the palm forest as a specific natural resource; this interpretation is different from B/T because the sector based on palm forest does not represent the production of all agricultural goods, which instead are produced in a traditional way with decreasing returns to scale. The presence of decreasing returns to scale allows us to assume the presence of a class of "landowners", which obtain a rent from the use of soil. This class can be thought of the possessor of technology, which is necessary for agriculture and fishing, as the evidence on Easter Island civilization supports.

The two classes employ the wood of palm forest for two different uses; the workers build canoes to catch fishes and therefore this resource sustains rapid population growth, while

[5] This social troubles are analysed by several authors, they have carried on a rich field of research based on the concept of appropriative competition, see for instance (Hirshleifer, 1995; Grossman and Mendoza 2003).

landowners utilize wood to transport the famous statues. In the model we do not consider these two final sectors as if workers and landowners consumed directly the wood (the harvest of the natural resource). The development of the Easter Island civilization needed a well-structured society, and the division of Island on clans confirms that there was a class holding the power.
To summarize, the first sector can be seen as an aggregation of all the subsistence activities, which are prevalently agricultural; while the other sector specifically develops around a natural resource (the palms in this case). Moreover, the harvest of palm forest was employed in two different uses, to get more food and to build and transport manufactures. It must be emphasized that workers utilize a specific technology for the exploitation of a particular resource and also to transport the "Moai". This is the reason why in the case of exhaustion of the natural resource stock, all the technology developed in that sector becomes unserviceable and therefore it gets forgotten. This reconstruction of the events also explains why "local residents had no knowledge of how to move the statues, and believed that the statues had walked to the platforms under the influence of a spiritual power" (B/T, p. 121).

III. Analytical model: income and distribution

The economy produces and consumes two goods. One is the harvest of a renewable resource, while the other is a traditional agricultural good named "corn"[6] that is produced with decreasing returns to scale with labour and land. We suppose that the land is abundant.
The production function of the harvest sector is the same of B/T (1998). The harvesting of the resources follows the Schaefer harvest production function (Schaefer, 1957), that is

$$H = \alpha S L_H , \tag{1}$$

where H is the quantity of harvest supplied by producers, S is the resource stock, L_H is the labor used in resources harvesting and α is a positive constant. This function underlines that the quantity of harvest produced is proportional to the stock of natural resource and to the quantity of labour applied; for this reason it is often called bi-linear. The parameter α represents in the literature on fisheries management the "catchability" coefficient, while in our model it represents the technological parameter. This very minimal function helps us simplify the analysis and also allows us to use the same production function of Pasinetti (1960) with an easy addition of the natural resource stock. We define $a_{LH}(S) = \frac{L_H}{H} = \frac{1}{\alpha S}$ the quantity of labor necessary in order to produce one unit of harvest. Assuming that there is perfect competition (therefore the profits are nil), the unit price of harvest must be equal to the cost of production, that is

$$p_H = w a_{LH}(S) = \frac{w}{\alpha S} , \tag{2}$$

where w is the monetary wage.

[6] We name the good "corn" to maintain the classical derivation of this model, but in fact the most common product of agriculture in Easter Island was sweet potato.

The traditional sector producing “broad” corn can be specified as follows:

$$C = F(L_C), \text{ such that } F(0) \geq 0,\ F'(0) \geq \overline{w},\ F''(L_C) < 0^{7},$$

where C is the quantity of corn produced, L_C is the number of workers employed in the production of corn, $\overline{w}$ is the natural wage (in a Ricardian sense). The Cobb-Douglas production function is one of the possible functions that meets these three properties; for simplicity we decide to specify the production function of this sector in this way

$$C = \lambda L_C^{\ \delta}, \qquad \text{with } 0 < \delta < 1 \text{ and } \lambda > 0. \tag{3}$$

Following Pasinetti (1960) we suppose that the landowners receive a rent equal to the sum of all the rents for each infra-marginal lands, that is "…a net gain for the owners of the more fertile lands with respect to the owners of the marginal land" (Pasinetti, 1960, p.83); therefore $R = F(L_C)p_C - L_C F'(L_C)p_C$. From equation (3) we obtain

$$R = \lambda(1-\delta)p_C L_C^{\ \delta}. \tag{4}$$

Setting corn as numeraire ($p_C = 1$), the condition of equilibrium in the corn sector is:

$$C = R + wL_C. \tag{5}$$

From equations (3), (4) and (5) we obtain:

$$w = \lambda\delta L_C^{\ \delta-1}. \tag{6}$$

This relation implies that the wage is, as expected, an inverse function of the number of workers employed in the corn sector.
We assume that landowners consume only the harvest good, therefore all the rent is spent in the harvest sector; while each consumer has an utility function of this type[8]:

$$u = h^{\beta}c^{(1-\beta)}, \tag{7}$$

where h and c are individual consumptions of harvest and corn, and β is a parameter such that $0 < \beta < 1$. This utility function stresses that the present generation does not care at all about future generations; its utility is given by the present consumption of resources. The maximization of the utility at a point in time, subject to the instantaneous boundary $p_H h + c = w$ yields $h = w\beta / p_H$ and $c = w(1-\beta)$. Since every consumer has the same preferences, total demand is equal to L multiplied by individual demand; but in the harvest sector we must add the quantity of harvest required by landowners. So we have

$$C^D = w(1-\beta)L, \tag{8}$$

[7] See Pasinetti 1960 p. 82.
[8] See B/T p. 123.

$$H^D = H_w^D + H_R^D = \frac{w\beta}{p_H} L + \frac{\lambda(1-\delta)}{p_H} {L_C}^{\delta}, \tag{9}$$

where the superscript *D* represents demand, and the subscript *w* and *R* represent the respective sources. At a moment in time, in order to have equilibrium in the corn sector, the supply of corn must be equal to the demand, hence from equations (3), (6) and (8) we have

$$C = \lambda {L_C}^{\delta} = C^D = w(1-\beta)L = \lambda\delta {L_C}^{\delta-1}(1-\beta)L,$$

therefore

$$L_C = \delta(1-\beta)L. \tag{10}$$

From (6) and (10) we have

$$w = \lambda\delta^{\delta}(1-\beta)^{\delta-1}L^{\delta-1}, \tag{11}$$

From (2), (4), (10) and (11), the quantity of harvest demanded by landowners can be written as

$$H_R^D = \frac{\lambda(1-\delta)}{p_H} {L_C}^{\delta} = \alpha(1-\delta)(1-\beta)SL.$$

Similarly to the corn sector the equilibrium of the harvest sector implies that

$$H = \alpha\beta SL + \alpha(1-\delta)(1-\beta)SL = \alpha(1-\delta(1-\beta))SL. \tag{12}$$

IV. Analytical model: equations of motion

The dynamics of the model are given by two equations; the former specifies the regeneration properties of the renewable resource *S*, the other represents the dynamics of the population. As will be clearer later on, the dynamics constitute a kind of Lotka-Volterra predator-prey model. The change in the stock of the natural resource at time *t*, $\dot{S}(t)$, is equal to the a function $G(S(t))$, representing the natural dynamics of the stock in a virgin state, minus the harvest rate $H(t)$. Following B/T, dropping the time variable, we specify the change of the stock without the intervention of men as $G(S) = rS(1 - S/K)$, where *r* is the regeneration rate and *K* is the maximum possible size of the natural resource. Therefore

$$\dot{S} = rS(1 - S/K) - H. \tag{13}$$

The dynamics of population follow a form of the Malthusian law of population

$$\dot{L} = L(w - \overline{w} + F), \tag{14}$$

where $\overline{w}$ is the natural wage (in this case the wage that implies $\dot{L} = 0$ when $F = 0$), $F = \phi \frac{H}{L}$ represents an externality given by the consumption of harvest on the dynamics of population, and ϕ is a positive constant. Higher per-capita harvest consumption leads to higher population growth. We decide to underline explicitly the role of wage to maintain our model as closest as possible to the Ricardian tradition. The main difference with respect to the basic model of Lotka-Volterra type concerns the possibility for the predator (the human population) to survive although the natural resource stock is exhausted. Indeed, the predator in our model chooses between two preys: one is renewable and exhaustible, while the latter (corn) cannot be depleted; we assume that this sector of subsistence is productive enough to feed a positive population ($\overline{L} > 0 : w(\overline{L}) = \overline{w}$). Before substituting the expressions of harvest in equations (13) and (14), we define

$$A := \lambda\delta^{\delta}(1-\beta)^{\delta-1}, \tag{15}$$

$$B := \alpha(1-\delta(1-\beta)), \tag{16}$$

where A and B are two positive constants. Therefore, from equations (11), (12), (15) and (16), the dynamic relations (13) and (14) become

$$\dot{S} = S[r(1 - S/K) - BL], \tag{17}$$

$$\dot{L} = L\left[AL^{\delta-1} - \overline{w} + \phi BS\right]. \tag{18}$$

V. Steady state and regimes

Given the equations of the dynamics of the model we are firstly going to analyse the steady state. There are four equilibria of our dynamical system:

i) $L = 0 \quad S = 0$

ii) $L = 0 \quad S = K$

iii) $L > 0 \quad S = 0$

iv) $L > 0 \quad S > 0$

The first equilibrium is an unstable point, while the second is an unstable saddlepoint allowing an approach along $L = 0$ axis. In the B/T model (i) is an unstable saddlepoint because there is not the third equilibrium. In our model it is possible to reach the steady state with positive population and zero stock of resource although the utility would be nil. Now we concentrate our attention on the last two equilibria because they are more interesting from an economic point of view.

We divide the quadrant $L > 0$ and $S > 0$ into sectors by the two lines:

$$E: \quad S = K - \frac{BK}{r}L,$$

$$P: \quad S = \frac{\overline{w}}{\phi B} - \frac{A}{\phi B} \frac{1}{L^{1-\delta}}.$$ [9]

There are two possibilities, according to whether or not these two lines intersect in the positive quadrant. Indeed from equation (18), the level of the population in the third equilibrium, that is when line P intercept the axis $S = 0$, is

$$L^* = \left(\frac{A}{\overline{\overline{w}}} \right)^{1/1-\delta}, \tag{19}$$

therefore, the two lines intersect in the positive quadrant if and only if

$$\left(\frac{A}{\overline{\overline{w}}} \right)^{1/1-\delta} < \frac{r}{B}, \tag{20}$$

that is, if and only if the intercept of the function $\dot{L} = 0$ on the population axis is less than the intercept of the function $\dot{S} = 0$. If condition (20) is not respected, equilibrium (iv) cannot exist. Therefore we can identify two different cases:

Case 1

The dynamics of the system when condition (20) is not respected are represented in figure 1[10].

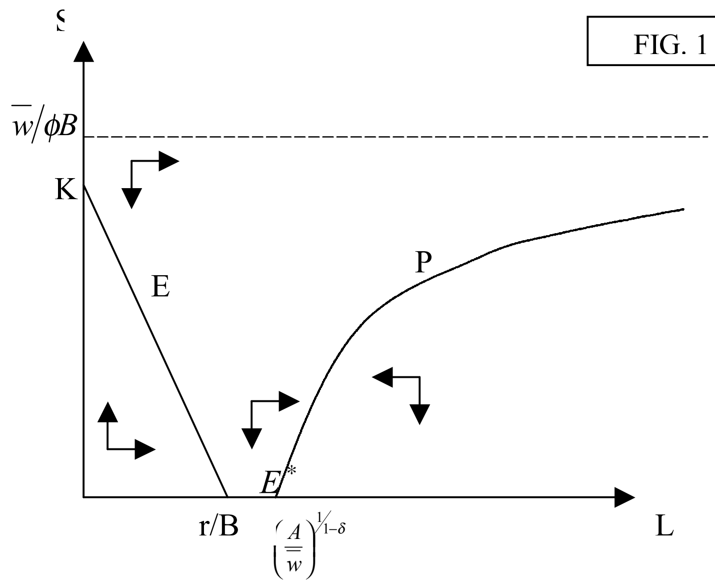

FIG. 1

[9] These two equations are obtained imposing $\dot{S} = 0$ $\dot{L} = 0$ when $L > 0$ and $S > 0$.

[10] In all the three figures we represent the asymptote of line P (with $S = \overline{w}/\phi B$) above the intercept of line E (that is *K*) without discussing the other case. But this condition does not influence the whole dynamics of the system.

In this first case, E^* corresponds to the steady state (iii) and it is asymptotically globally stable: the resource is exhausted and the population approaches its limiting value $\left(\frac{A}{\overline{w}}\right)^{1/1-\delta}$.

The easiest explanation is that the rate of regeneration of the natural resource is too low. If the population is on the left of its limiting line it must increase, and after a while the natural resource stock starts to decreasing (when the trajectory crosses line E). After that point the natural resource can never increase, while the population continues to increase. The trajectory approaches the line P even more slowly, because close to that line the dynamics of population tends to be stable, therefore in the limit the trajectory will approach E^* in the neighbourhood of line P, that is in the state of the economy fully described by the corn sector. Point E^* will not be reached in finite time, but it is interesting to notice that we tend to E^* even if the price of the resource tends to infinite. The increase of price is not capable of leading the economy to a sustainable equilibrium.

Case 2

When condition (20) is respected, functions (21) and (22) can be represented in figure 2.

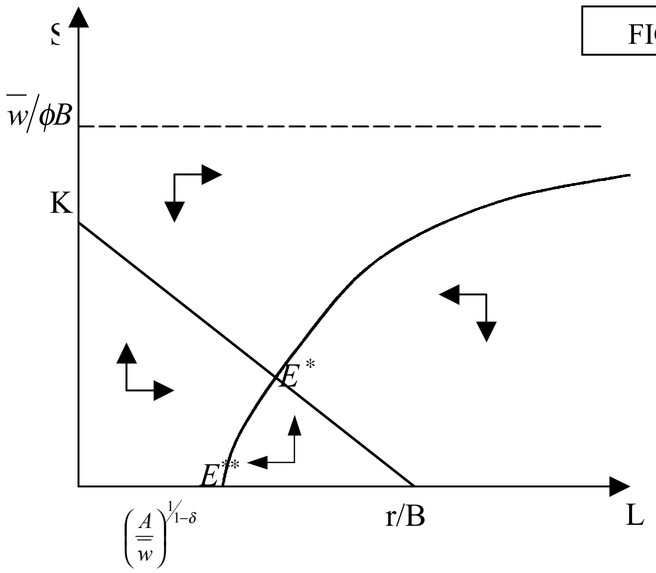

FIG. 2

The second regime is characterized by a locally asymptotically stable equilibrium with positive resources and positive population (the proof of local stability is given in the appendix 1).

We can make some remarks also about global stability. Let us suppose that the system is in a virgin state with $L = 0$ and $S = K$. Now we assume that a small group of people arrive at that island. They find a very comfortable environment and the population increases very fast. The number of people in the first period is irrelevant for the dynamics of the natural resource stock, but gradually the population starts degrading the renewable resources. There will be an instant when the dynamics of population will start slackening and the system will approach the line P (see figure 3) in the point *p*. From *p* the population will start decreasing, first slowly and then

faster. The resource continues to degrade but even more slowly. Given the dynamics of the natural resource it is impossible in finite time to reach the $S = 0$ axis (see appendix 2).
Therefore, the system will meet the line E, while the population continues to decrease. We are now in the III quadrant, the population is again low enough to allow the natural resource to regenerate. In the neighbourhood of line P the population is almost stable and the natural resource stock grows. Very soon the environment will be capable again of allowing an increase of population. In the IV quadrant both population and the resource stock can grow and the natural resource dynamics will be in equilibrium on line E at the point *e*. This example clarifies that from any starting point with $L > 0$ and $S > 0$ the dynamics of the system can neither be explosive, nor reach one of the two axes in finite time.

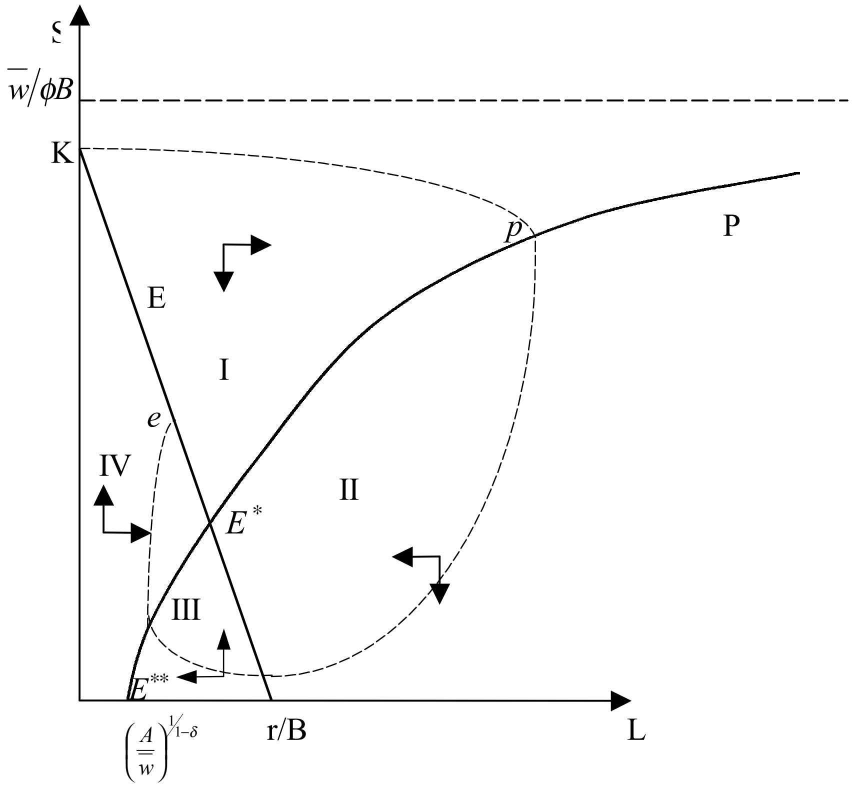

FIG. 3

Proposition 1

Given that E^* is locally stable (see appendix 1), and that in quadrant II the trajectory cannot reach in finite time the $L = 0$ axis (see appendix 2); any trajectory cannot enter in the basin of attraction of another equilibrium. Therefore, E^* is globally stable if we exclude from the initial condition the two axes; that is if we only consider the cases in which both $L(0) > 0$ and $S(0) > 0$ are respected.

VI. The role of technical progress

Until now we have never considered the role of technical progress in this model. As we pointed out in the introduction, in the Ricardian system technical progress contrasts the reduction of profits and therefore changes the speed to which the economy approaches the steady state. We can also suppose that thanks to technical progress the economic system never reaches the steady state, but technical progress can neither change the functioning of the system, nor the features of the steady state. This is the reason why Pasinetti (1960) in his mathematical formulation of the Ricardian system considers the dynamics of the economy without technical progress. But in this model the effects of technical progress can induce a change of the system from one to the other regime.

Suppose that we are in the second regime where condition (20) is respected. Let us consider how the effect of an exogenous increase of the technological parameters can change the lines P and E in figure 3.

When α increases, the intercept of line E with the $S=0$ axis decreases; while when λ increases the curve P shifts towards the right. Therefore if α and λ increase exogenously, in the long period the system switches from the second to the first regime. We can derive a simple explanation for this result: the effect of an increase in α is an increase in the harvest, therefore the population which is sustainable with a positive natural resource stock decreases (in the graph *r/B* decreases). In the case of an exogenous increase of λ the sector producing corn increases its output, therefore the wage increases (see equations 6 and 11) and the equilibrium level of population with $S=0$ increases al well.

There is another technical parameter that is δ, but the effects of its increase are less clear. Indeed in this case from one side the intercept of E on the $S=0$ axis increases, while the P line translates towards the right and the asymptote shifts downwards. Without imposing strong restrictions on the values of the other parameters it is impossible to show the aggregate effect of a change of this parameter.

However the effect of a change of the two first parameters (α and λ) on the dynamics of the system is totally clear. The improvement of the technology, through α and λ, makes the shortage of palm forest deeper, therefore technical progress in our model cannot help to solve the problem given by the absolute scarcity of a crucial resource. This result was simply perceived by Reuveny and Decker (2000) through a number of simulations in an extension of B/T model, they assert "in the majority of our simulations, the population and the resource fluctuated widely over time; technological progress would not necessarily have generated a 'golden path' of economic growth".

VII. Preferences

The preferences are resumed by the parameter β that in the dynamics is hidden behind the constants *A* and *B*; in particular β represents the elasticity of utility respect to the per capita harvest. From equations (15) and (16), an increase of β increases both *A* and *B*; and, as expected, it increases also the depletion of the natural resource stock[11].

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[11] The increase in *A* has the effect of translating line P towards the right, while when *B* increases, the intercept of line E with the axis $S=0$ decreases.

This simple representation of preferences is still useful to compare different path of development. If two societies have two comparable productive and social structures, the difference of preferences can be the chief element to explain two completely different paths of development.
In a society where the preference for harvest is quite strong (that is when β is high enough) the growth of civilization is quite fast but probably not sustainable: after a (more or less long) period of grandeur this society is destined to experiment a profound crisis; while a society with low preference on the harvest of the natural resource stock could gradually reach sustainability around the steady state with positive and constant utility.
It is clear that when the resource starts becoming scarce there is more room for troubles; the differential equations are not suitable to explain this sort of discontinuities, for example the people can either fight to conquest the remaining resource stock or they can faster destroy it. Therefore there are several reason to believe that when the equilibrium E^* of figure 2 is close to E^{**} it is easy to switch from the second to the first regime. The history of Easter Island seems to support this reasoning, there are historiographic and geological proves that while palms were exhausting there were, for the first time, proves of several acute fights inside the Easter Island society.

VIII. Concluding remarks

One of the main weaknesses of that model is that it does not analyse the effect of deforestation on the ecosystem of Easter Island. Erosion of soil, reduction on the availability of water, extinction of birds and other species are just few examples of what followed of the destruction of palm forests. Moreover since ecological systems are characterized by thresholds, multiple equilibria and irreversibility, it is very plausible that when the exploitation arrived at a certain level the condition to attain sustainability could not be reached anymore. These conjectures increase the possibility that Easter Island civilization flipped from the second to the first regime where a long period position with positive resource stock and utility is not sustainable. Differently from what the economic literature on Easter Island asserts, this more resilient final equilibrium admits positive population, a result that was found by the first Spanish explorers. These survivors continued to use the traditional sector that was productive enough to maintain alive a small population. From the other side, the specific technology used in all the process based on palm forest exploitation (hence also how to transport the "Moai") became completely useless and therefore was forgotten.
One of the most important results of this paper is that the explanation of the history of Eastern Island civilization has been obtained coherently with the classical analysis, through a simple manipulation of the Pasinetti's formulation of the Ricardian system. In this way we have proved that Ricardian models of growth can be a useful instrument to analyse problems of co-evolution between population and natural resources, and more generally, the relation between economic growth and ecological sustainability.

IX. Appendix

1. Local stability

The steady state *E*=(S*, L*)* is the solution of the system

$$\begin{cases} r(1-S/K)-BL=0 \\ AL^{\delta-1}-\overline{w}+\phi BS=0 \end{cases}, \tag{1.A}$$

We can proceed linearising the dynamical system around E* using Taylor series expansion for the vector $e=(e_S,e_L)=(S-S^*,L-L^*)$ around $e=0$. We obtain

$$\dot{e}=\mathrm{J}(S^*,L^*)\cdot e+R(S,L),$$

where **J** is the Jacobian matrix of first order partial derivatives with respect *S* and *L* evaluated at (S^*,L^*) that is

$$\mathrm{J}=\begin{pmatrix} r-2rS^*/K-BL^* & -BS^* \\ \phi BL^* & -\overline{w}+\phi BS^*+\delta A(L^*)^{\delta-1} \end{pmatrix},$$

and the trace of **J** is

$$\mathrm{Tr}(\mathrm{J})=(r-2rS^*/K-BL^*)+\left(-\overline{w}+\phi BS^*+\delta A(L^*)^{\delta-1}\right).$$

We can compare this last equation with the system (1.A): from the first equation the first parenthesis of the right side of trace is equal to $-rS^*/K$ because the other members are zero (see firs equation of system 1.A); from the second equation and the second parenthesis rest only $-(1-\delta)A(L^*)^{\delta-1}$. We obtain

$$\mathrm{Tr}(\mathrm{J})=-rS^*/K-(1-\delta)A(L^*)^{\delta-1}<0,$$

therefore the real part of the characteristic equation is always negative and the equilibrium is locally asymptotically stable.

2. Approaching axis S=0

We are going to prove that starting from any initial point in the II quadrant (see figure 3), the trajectory of the dynamical system crosses line E and therefore it can never reach the $S=0$ axis. Considering the dynamics of the resource stock (see equation 17), in the II quadrant the population decreases and this reduces the velocity of the decrease of natural resources stock. Therefore if we consider the population constant we overestimate the velocity of the dynamics. We prove that even in this case the natural resource stock approaches $S=0$ only when the time tends to infinite. The dynamics of natural stock can be written as

$$\dot{S}=(r-BL_o)S-r/K\,S^2=-\overline{L}S-r/K\,S^2,$$

where L_o is the initial condition and $\overline{L}=(r-BL_0)$ is negative because we consider that initially $L_0>r/B$ that is the population tends to decrease the natural resource stock. Solving the differential equation, we find that all the coefficients of the characteristic equation are positive, therefore whatever is the sign of the determinant, the natural resource stock tends to zero if and only if the time tends to infinity. The trajectory passing from L_o crosses line E before reaching the $S = 0$ axis and arriving at the third quadrant.

X. References

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